Masters in
Environmental Engineering
Energy Engineering and Management
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Executive Master in
Sustainable Energy Systems

Doctoral Programs in
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Environmental and Natural Resource Economics
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Energy and Environment
Sustainable Development, Energy and Environment
Theory and Practice of Sustainable Development

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Problem Set

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Chapter 1 – Apartment Market Example

Consider the following problems within the context of apartment market studied in theoretical classes.

1) Suppose we have 25 people whose reservation prices equal 500 €, and that the 26th person equals 200 €. Draw the demand curve.
2) In the previous example, what would be the equilibrium price, if there were 24 apartments for rent? What if there were 26? Or 25?
3) Consider a situation where part of the apartments was converted from rent to sale (i.e., the situation discussed in section 1.6 of Varian, 1987). Suppose each apartment for sale was built from two apartments for rent. What would happen to the apartments' price?
4) What do you think would be the long-term effect of a tax imposed on the number of apartments built?
5) Suppose the market is formed by 8 people, whose reservations price (expressed in €/day) can be found in the following table:

Person	A	B	C	D	E	F	G	H
Price	40	25	30	35	10	18	15	5

   a) Draw the demand curve.
   b) With a supply of 4 apartments, who would end up getting the apartments?
   c) Assuming the supply increases to 6 apartments, what is the range of equilibrium prices?
6) Consider the situation in problem 5) and assume that a monopolist owns all the apartments.
   a) Calculate the maximum revenue he can make when renting 1, 2, ..., 8 apartments (assuming he is a non-discriminative monopolist). If the monopolist has 8 available apartments, how many does he decide to rent, and for what price?
   b) Apply the previous point to a discriminative monopolist.
7) In theoretical classes we considered the situation where a tax was imposed on the landlords. Now consider the tax was instead imposed on the renters.
   a) Answer problem 5) applying this new situation.
   b) Generalise to an arbitrary situation a conclusion on the effect of a tax imposed on renters.

Chapter 2 – Budget Constraint

1) Manuel was consuming 100 units of X and 50 units of Y. The price of X increased from 2 to 3. The price of Y remained 4. How much does Manuel's income need to increase so he can afford the same 100 units of X and 50 units of Y?
2) (EA exam of 25.05.2006). State in which of the following cases the budget constraint line moves away from the origin without changing its slope.
   a) The prices of X and Y increase 15%.

b) The price of X decreases 10% while the price of Y increases 10%.

c) The prices of X and Y increase 5% while income decreases 5%.

d) The prices of X and Y decrease 10% while income increases 10%.

Chapter 3 – Preferences

1) Consider a group of people A, B and C, and the relation "at least as tall as". Is this relation transitive? Is it complete?

2) Consider the same group of people and the relation "strictly taller than". Is this relation transitive? Is it reflexive? Is it complete?

3) A football coach says that given any two players, he always prefers the one which is taller and quicker. Is this preference relation transitive? Is it complete?

4) (Exam of 25.05.2006). Suppose Lucia is given the possibility of choosing between "a trip to Mozambique and three months' pass to Expo 98" and "three trips to Mozambique and a one month's pass to Expo 98". Say which of the following answers violates the axioms and hypothesis that rule preferences:

   a) "They are so different, I can't choose"

   b) "I don't mind, you choose it for me"

   c) "I know I'll regret any bundle I choose".

5) (1st Season Exam of 2004-05). Joana likes chocolate cake and ice-cream, but after eating 10 slices of cake, she is full of it, and eating more chocolate cake leaves her less satisfied. Joana always prefers to eat more ice-cream than less. Sketch indifference curves that are consistent with this description.

6) (2nd Season Exam of 2004-05). Joana likes chocolate cake and ice-cream, but after eating 10 slices of cake, she is full of it, and eating more chocolate cake leaves her less satisfied. Joana always prefers to eat more ice-cream than less. Joana's parents allow her to leave aside everything she dislikes eating. Sketch indifference curves that are consistent with this description.

7) (Special Season Exam of 2004-05). Thomas is happiest when he eats 8 cookies and 4 cups of milk per day. Whenever he has taken those quantities, giving him more of either leaves him less happy. When he has taken less than those quantities, giving him more of either leaves him happier. His mother makes him drink 7 cups of milk daily and only allows him to eat 2 cookies. One time his mother was out, Tomas' sadistic sister forced him to eat 13 cookies and only gave him one cup of milk, ignoring his bitter protests about the last 5 cookies she gave him and his begging for more milk. Even though Tomas complained to his mother about his sister's behaviour, he had to admit he preferred his sister's forced feeding to that of his mother's. Sketch indifference curves consistent with this description.

8) (EA Exam 13.01.2006). Consider a course where the mark, ranging from 0 to 20, is given by the continuous evaluation mark, weighting 20%, and the mark of the exam, weighting 80%. Show the indifference curves for the students of this course, as a function of the continuous evaluation grades and the exam grades.

9) (EA Exam 25.01.2006). Consider a course where the mark, ranging from 0 to 20, is given by the continuous evaluation mark, weighting 20%, and the mark of the exam, weighting 80%. Furthermore, to pass the course, students need to have a minimum mark of 9.5 mark in the exam. Show the indifference curves of the students of this course, in relation to the continuous evaluation marks and the exams marks.

10) (EA Exam 08.01.2008) Graphically represent the preferences of the following consumers:

   a) Gonçalo always drinks a coffee with a glass of water.

   b) Graça is indifferent between using A4 ruled paper or A4 plain paper.

   c) Maria cannot eat more than 220 g of meat but drinks all the Coke that is served to her.

11) (EA Exam 12.01.2009) Consider the following preferences:

$(x_1, x_2) \succ (y_1, y_2)$ iff $x_1 > y_1$.

   a) Say, justifying as much as possible, whether these preferences are:

      i) complete;

      ii) reflexive;

      iii) transitive;

      iv) monotonous;

      v) convex;

   b) Represent qualitatively this family of indifference curves.

12) (EA Exam 20.01.2011) Suppose that in Springfield there are two types of food, meat and fish. Draw, justifying, plausible indifference curves for Homer, Lisa and Flanders. Remember to use arrows to depict the direction of increasing utility.

   a) Homer prefers variety and likes to eat meat and fish together.

   b) Lisa is a vegetarian and doesn't like meat or fish.

   c) Flanders loves to eat and enjoys variety, but he also wants to control his weight.

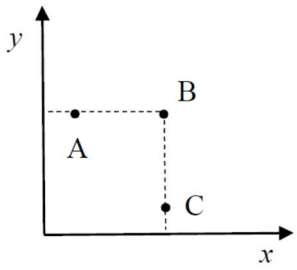
13) (EA Exam 18.01.2016). Sketch a plausible family of indifference curves, for bundles of (environmental quality, GDP), for the following situations:

   a) a diehard environmentalist, who only cares about the environment;

   b) someone who only cares about economic activity and does not care about the environment;

   c) someone who cares about both the environment and material affluence but considers it unacceptable to go below a certain threshold of environmental quality.

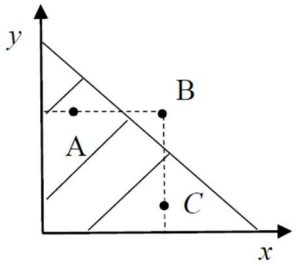
14) (EA Exam 02.02.2016) Consider a market where there are only the three consumption bundles (x, y) presented in the following graph:



a) Consider consumer Y with the following preferences: strictly prefers A to B; strictly prefers B to C; strictly prefers A to C. Are his preferences transitive? Are they complete? Are x and y goods or bads for this consumer? Sketch a family of indifference curves compatible with these preferences.

b) Consider consumer X with the following preferences: strictly prefers A to B; strictly prefers B to C; strictly prefers C to A. Are his preferences transitive? Are they complete? Sketch a family of indifference curves compatible with these preferences.

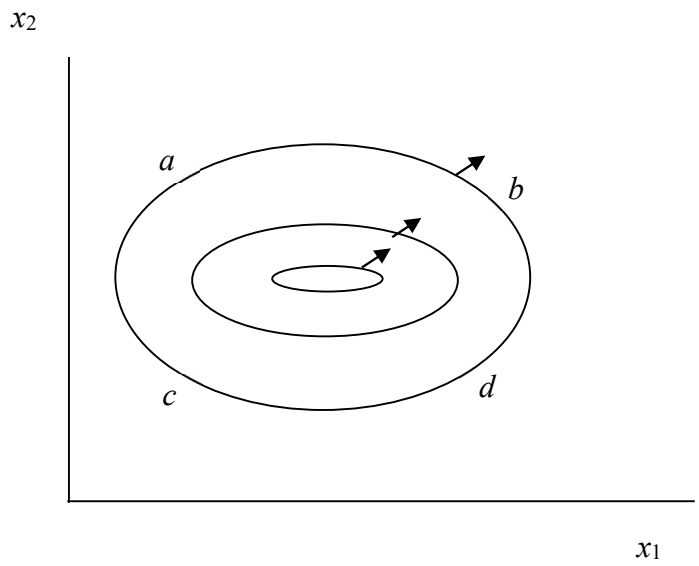
c) Suppose consumer Y has the budget constraint $p_1x + p_2y \leq m$, graphically presented in the following figure. Which consumption bundle does he choose?



15) (EA Exam 01.02.2017) Give real world examples of situations in which the behaviour of consumers violates the properties of:
   a) completeness;
   b) reflexivity;
   c) transitivity.

Chapter 4 – Utility

1) (EA Exam 12.01.2007). Consider the family of indifference curves shown in the graph below. For each of the points a, b, c and d, state the signal of the marginal rate of substitution and of the marginal utilities of goods 1 and 2.



Chapter 5 – Choice

1) (Exam of EA 13.01.2006). Albert has 50 € and consumes goods *X* and *Y*. His utility function is represented by: $U = (x+15)^2 y$ (where x and y are the consumed amounts of goods *X* and *Y*, measured non-dimensionally), and the prices of *X* and *Y* are, respectively, 3 € and 5 €.

   a) Calculate the consumption basket chosen by Albert.

   b) Suppose Albert is presented the possibility of winning a prize in quantity (5 units of X, which he cannot sell) or 15€ in cash (the cost of 5 units of X). Which prize would Albert prefer? Justify analytically.

2) (Exam of EA 12.01.2007). Jeremiah is five years old. He likes to eat chocolates but hates to eat spinaches; moreover, the more spinach he eats, the more he hates it. His mother lets him eat two chocolates every day, and for each 100 g of spinaches eaten, he is allowed to eat one more chocolate.

   a) Sketch qualitatively a set of indifference curves consistent with the information you have on Jeremiah's preferences.

   b) In the same graph, represent the constraint Jeremiah is subject to.

   c) Show in the graph the optimal consumption point of chocolates and spinaches that Jeremiah chooses.

3) (Exam of EE 11.07.08) Consider a consumer with the utility function

$$u(x,y) = kx^a (y+\theta)^b,$$

   where *k*, *a*, *b* and θ are positive constants. Considering that the consumer has the budget constraint

$$p_1 x + p_2 y = m,$$

   a) calculate the demand curves for each good,

b) depict graphically and interpret the effect of income on the optimal quantity consumed of y,

c) interpret the effect of p_2 on the preceding result.

4) (Exam of EA 25.01.2006). Ti Manel can be considered an example of an average consumer of a certain community. In what regards the consumption of goods "KULTURA", K, e "COPUS", C, his utility function is $U = (K + 100)(C + 100)$, where K and C are measured in non-dimensional quantities. On average, Ti Manel allocates 25 € to the consumption of those goods, whose unit prices are 3 € and 2 €, K e C respectively. In spite of the budget constraints imposed by the Minister of Finance, the government decided to grant the consumers of Ti Manel income level, a subsidy of 25% over the price of K. Determine Ti Manel's consumption basket, before and after the implementation of that measure, discussing the obtained results.

5) (EA Exam 19.01.2004). Consider the utility function $U = \ln(X_1 X_2)$, where U is the utility and X_1 and X_2 are, respectively, the consumption of good 1 and good 2.

a) Draw in a graph the curves of constant utility, in the plane (X_1, X_2).

b) Goods 1 and 2 are perfect substitutes, perfect complements, or none of these? Justify.

c) Consider a consumer whose behaviour is described by this utility function. Suppose the price of good 1 is p_1, the price of good 2 is p_2, and the consumer's income is I. Establish, according to the problem's parameters, the quantities of X_1 and X_2 that the consumer will choose.

6) Consider the Cobb-Douglas utility function. Calculate the consumed amounts of goods 1 and 2 by a consumer subject to the budget constraint $p_1 x_1 + p_2 x_2 = m$.

7) (Exam of EA 12.01.2005) Escola Secundária do Monte da Amora (ESMA) has 60 000 € to spend on computers (the expense on computers is given by C) and other things (expenses on other things is given by X). The Ministry of Education wants to encourage "computer science literacy" and so proposed the following plans:
Plan A: The government grants a 10 000 € subsidy to each school to spend as they prefer.
Plan B: The government grants a subsidy equal to 50% of the expenses of each school in computers.
Plan C: The government grants a subsidy equal to 50% of the expenses of each school in computers, up to a maximum of 10 000 € subsidy.
Plan D: The government grants a subsidy equal to 10 000 €, to be spent exclusively on computers.

a) Write the budget constraint equations of ESMA for each of these plans.

b) Suppose ESMA has preferences whose utility function can be represented by $U(C, X) = CX^2$. Determine optimal expenses of ESMA with computers, for plans A and C. Interpret the difference between results.

8) (Exam of EA 13.09.2005). Ambrósio likes to eat nuts and blackberries. His utility function is $4\sqrt{x_1} + x_2$, where x_1 is the amount that consumes nuts and x_2 is the amount that consumes blackberries (both expressed in kg). The price of nuts is 1 €/kg and the price of blackberries is 2 €/kg.

a) Sketch indifference curves for Ambrosio's utility function.

b) Suppose Ambrósio has 24 € to spend. How much nuts and blackberries does he decide to buy?

c) Suppose now that Ambrósio only has 9 € to spend. How much nuts and blackberries does he then decide to buy?

9) (EA Exam 12.01.2007). David works to earn his income, receiving payment for each working hour. David enjoys his leisure time but does not enjoy working. On the other hand, he likes to consume several goods, for which he needs remuneration from working. Therefore we assume David's utility function is one of the type $U(C,L)$, where C is the amount spent in consumption and L is the leisure time (working time is then 24h-L). David's wage is w (in €/h) for the time he works less than eight hours a day and w' for the time he works over eight hours a day, with $w' > w$. Suppose David's utility function is given by $U(C,L) = CL^2$. Determine David's choice in terms of working time.

10) (EA Exam 09.02.2008). Belito is a 7-year-old boy whose aunt has given him x books and y videogames. The cost of the books is p_1 and that of videogames is p_2. No exchanges are permitted after purchasing the goods. Belito's utility function is: $U(X,Y) = X\,Y^6$, where X and Y are respectively the number of books and videogames that he has. Determine the value of the maximum utility Belito could attain, if his aunt gave him in cash the total amount of the gifts, instead of giving him the gifts.

11) (EA Exam 09.02.2008). Consider a consumer with Cobb-Douglas utility, $U(x_1,x_2) = kx_1^a x_2^b$, where a, b and k are positive constants.

a) Suppose the consumer wants to reach a level of U^* utility. Determine the minimum cost he must bear for that, considering that the prices of goods 1 and 2 are respectively p_1 and p_2.

b) Interpret the effect of the problem's parameters on the solution.

c) Draw a graph of the problem's solution.

d) In analogy with what is done for producer theory, re-write the utility as a cost function.

12) (Exam of EE 11.07.2008) Vasco expresses his preferences according to the following utility function:

$$U(L,C) = LC + L + 2C,$$

with budget constraint $p_1 L + p_2 C = m$.

Determine Vasco's demand curves for goods L and C. Interpret results.

13) (Exam of EE 20.01.2011) Consider a consumer whose preferences are represented by the quasi-linear utility function $u(x,y) = \ln(x) + y$, where x and y are non-dimensional (i.e., they are measured as number of goods). The available income is known.

a) Write down the consumer's utility maximization problem.

b) Draw the budget constraint and representative indifference curves on the same graph.

c) Find the consumer's optimal consumption bundle (x^*, y^*) and depict the situation on a graph.

d) Represent in a graph how the consumption of each good changes when changing the prices of each good.

e) Interpret the effects of the prices and income on the consumption of each good.

f) What are the shares of income spent on each good? Interpret.

14) (Exam of EA Special Season) Consider that the utility function of a Portuguese person is given by $U(C, A)$, where *C* is an aggregate consumption measure of the goods traded in the market, and *A* represents the use of environmental goods available in Portugal. Discuss whether this function should have a perfect complements, perfect substitutes, imperfect substitutes or imperfect complements shape.

15) Vasco's preferences for goods v and u are expressed by the following utility function:

$U = k \ln\left[(v-b)\left(u+\frac{a}{v}\right)\right]$, with budget constraint $p_1 v + p_2 u = m$, where m is income.

Determine Vasco's demand curve, or the inverse demand curve for good u (you can determine quantity of good u as a function of price function, or the price of good u as a function of quantity). Interpret the results.

16) (Test EA 23.11.2011) Consider a consumer with the utility function:

$$u(x, y) = \ln\left(x^2 y\right)$$

The consumer has the budget constraint

$$p_1 x + p_2 y = m,$$

where *m* is the available income.

a) With this utility function, is the consumer willing to substitute a 1% decrease in *x* by a 1% increase in *y*? If not, by how much increase in y?

b) Calculate and draw the demand curves for *x* and *y* that maximize the utility of the consumer subject to the budget constraint. Interpret the changes in demand due to changes in prices and available income.

17) (Env. Econ. Exam, 18.01.2014) Consider a consumer whose behaviour is described by the utility function $u(x, y) = (ax + by)^2$

a) Calculate the marginal utilities.

b) Calculate the marginal rate of substitution. Interpret.

c) Sketch the family of indifference curves corresponding to this utility function. Suppose now the consumer faces the budget constraint $p_1 x + p_2 y = m$.

d) Represent graphically the budget constraint and the optimal point.

e) Formulate the Lagrangean for this optimisation problem and obtain the candidate interior solution(s); if necessary, calculate the optimal corner solution.

18) (Env. Econ. Exam, 18.01.2012). Consider the utility function

$$U(C,P)=\frac{C^{1-\sigma}-1}{1-\sigma}-\kappa\frac{P^{\phi}}{\phi},$$

where C is consumption, P is pollution, σ, κ and ϕ are positive constants, and $\sigma < 1$. This utility function is used to analyse the trade-off in utility between consumption and pollution. It was used, for example, in the energy efficiency game CityOn (http://www.cityon.pt/), developed by Biodroid and IST for EDP.

Consider $\phi = 1$.

a) Calculate the marginal utilities of consumption and pollution. Interpret.

b) Calculate the MRS. Interpret. Namely, consider the effect on C.

c) Sketch the family of indifference curves corresponding to this utility function.

Suppose now that there is a trade-off between spending income on consumption or on abating pollution, given by $C+a(P_o-P)=m$, where a are unit abatement costs and P_o-P is the amount of reduced pollution, where P_0 is pollution in the absence of abatement and P is the pollution effectively suffered by consumers.

Assume $m < aP_0$.

d) Interpret the last condition. Give real life examples where it makes sense and where it does not.

e) Sketch this budget constraint on the graph drawn in c), noting that, necessarily, $0 \le P \le P_0$ and $0 \le C \le m$.

f) Determine graphically the optimal solution, considering, if relevant, the different qualitative possibilities. Interpret.

g) Determine analytically the optimal solutions. Interpret.

19) Consider a society with aggregate preferences for consumption, C, and environmental quality, Q, with these aggregate preferences described by a utility function $U(C, Q)$. Suppose that these preferences are such that this society always prefers to, other things being equal (*ceteris paribus*),

- have more consumption than less, but the benefit of increasing consumption decreases with consumption;
- have more environmental quality than less;
- a combination of consumption and environmental quality instead of each by itself;
- can survive with environmental quality but cannot survive just with consumption.

a) Sketch a family of indifference curves consistent with this description.

Consider now that this society faces a trade-off between C and Q, because it uses dirty production processes which imply that, for each unit of C, there is a degradation of Q equal to ϕ. Suppose that in the absence of human activity environmental quality is equal to Q_0.

a) Write and represent graphically this constraint.

b) Represent graphically the optimal choice for this society. Interpret.

Suppose that this society invests in environmentally friendly technological innovation, and becomes more efficient, its production process now being described by ϕ'.

c) What is the relation between ϕ' and ϕ? What is the effect of this change on the optimal solution?

d) Consider now a long-term change in ϕ, associated to technological progress along humanity's development, with ϕ going from a very high value to a very low value. Sketch the change in environmental quality along time, assuming that at each instant t this society makes an optimal choice of C and Q. Interpret.

20) (EnE Exam, 27.01.2014) There are many different ethical theories about how resources should be distributed among individuals than can be described by considering a "utility function" for society (more precisely called a "social welfare function"), which is a function of the allocation of resources to each individual. Here we will study what may be called a "Nietzschean" view of society, where priority is given to the individuals that can make the most out of the resources they get.
To model this, consider, for simplicity, a society with just two individuals, where the "utility" of society is given by $u(x_1, x_2) = \max(ax_1, bx_2)$, with x_1 the amount of resources allocated to individual 1 and x_2 the amount of resources allocated to individual 2. There is an overall resource constraint given by $x_1 + x_2 = X$, where X, a constant, is the total amount of available resources.
Note that the utility function can be represented as a function with branches:
$u(x_1, x_2) = \begin{cases} ax_1 & \text{for} \quad ax_1 \geq bx_2 \\ bx_2 & \text{for} \quad ax_1 < bx_2 \end{cases}$.

a) Sketch the family of indifference curves for this utility function.

b) Represent graphically the optimal solution. Interpret.

c) Formulate the Lagrangian and calculate the candidate interior solution(s). If necessary, calculate the corner solution(s) and find the optimal one. Interpret.

21) (EA Exam 17.01.2015) Consider the utility function, $u(x, y) = ax + b \ln y / y_0$, where a, b and y_0 are positive constants.

a) Draw the family of indifference curves for this utility function.

Assume now that a consumer's preferences are described by this utility function and that she is subject to the budget constraint $p_1 x + p_2 y = m$.

b) Calculate the x^* and y^* that maximise his utility.

c) Sketch the graphs of x^* and y^* as functions of p_1 and p_2 (four graphs in total).

d) Interpret the results.

22) (EA Exam 18.01.2016) Consider an isolated thermodynamic system divided in two parts, (1) and (2), by an adiabatic wall, with different temperatures, $T^{(1)}$ and $T^{(2)}$, and internal energies, $U^{(1)}$ and $U^{(2)}$.
Thermodynamics tell us that, if we remove the insulation in the adiabatic wall, the system will evolve to a new equilibrium, which maximises the total entropy, $S = S^{(1)} + S^{(2)}$, under the constraint of energy conservation: $U^{(1)} + U^{(2)} = U^*$. The problem is analogous to that of a consumer maximising utility (*cf.* Sousa and Domingos, 2006a,b), under the budget constraint $p_1 x_1 + p_2 x_2 = m$ (the only difference being that the energy "prices" are equal to 1).
Suppose each side of the system has a perfect gas, with entropy given by:

$$S^{(1)} = N ln\, U^{(1)}/N, \quad S^{(2)} = N ln\, U^{(2)}/N,$$

where N is the (equal and constant) number of moles on each side of the wall.

a) In the ($U^{(1)}$, $U^{(2)}$) plane, draw the budget constraint for the thermodynamic system.

b) In the ($U^{(1)}$, $U^{(2)}$) plane, draw the "indifference curves" for the thermodynamic system, i.e., the curves where total entropy is constant.

c) In the ($U^{(1)}$, $U^{(2)}$) plane, indicate graphically the equilibrium point after the insulation is removed, i.e., the point of maximum total entropy compatible with the total internal energy in the system, U^*.

d) Using the method of Lagrangian multipliers, calculate $U^{(1)}$ and $U^{(2)}$ after the insulation is removed, i.e., calculate the values of $U^{(1)}$ and $U^{(2)}$ that maximise the total entropy, given the energy conservation constraint.

e) Calculate the "marginal utilities" of the system. What is the relation between the two at equilibrium? Given the physical meaning of an equilibrium reached after removing the insulation in an adiabatic wall, what do you think it the physical meaning of these "marginal utilities"?

23) (EA Exam 01.02.2017) In this problem, we are going to model the daily trade-off between labour and leisure. We start by considering the utility function given by $U = kC^a l^b$, where C is consumption and l is leisure. Consider now that consumption is equal to total wage obtained, i.e., $wL = C$, where w is salary per unit time and L is work (i.e., time worked). We can then write the utility function as:

$$U = k(wL)^a l^b$$

a) Sketch the family of indifference curves for this utility function, in the (l,L) plane.

b) Calculate the marginal utilities for labour and leisure and the marginal rate of substitution between the two. Express verbally the meaning of these concepts and interpret their specific expressions for this case.

Consider now the time constraint, $L + l = 24h$.

c) Represent this constraint on the graph in question a).

d) On the same graph, represent the optimal allocation of time to labour and leisure.

e) Calculate the optimal allocation of time between labour and leisure.

f) Interpret the result, including a discussion on whether there is a corner solution and what its meaning is (or would be).

24) (EARN Exam 17.01.2020) Vera Guedes has decided to go on a diet after the New Year. She is following a two-food diet, meaning that she can only eat two types of food items: eggs (E), and potatoes (P). Vera's preferences regarding her diet are described by the following utility function (with c and d positive constants and $c > d$):

$$U(E,P) = (c \cdot E + d \cdot P)^{1/2}$$

a) How much satisfaction does Vera get from consuming one more egg? And one more potato? What is the marginal rate of substitution between the two food items? Interpret your results.

b) Sketch the family of indifference curves for Vera's utility function. According to her preferences, are eggs and potatoes perfect substitutes, perfect complements, or none of these? Explain.

At the market where Vera does her weekly shopping, the price of eggs is p_E and the price of potatoes is p_P. Her food budget is given by m, which she spends exclusively on eggs and potatoes.

c) Write the budget constraint for this situation. Sketch the budget constraint in relation to the indifference curves, for all possible cases. Identify a corner solution graphically. (Note: you can draw several graphs).

d) Solve Vera's optimization problem. Write your candidate interior solution(s) and corner solutions.

Suppose that Vera decides to switch one of her two food items, potatoes, for sweet potatoes S, which are sold at price p_S. Her new utility function is given by $U(E,S) = E + \ln S$.

e) What are now the optimal quantities of eggs and sweet potatoes that maximize Vera's utility?

f) Based on your calculations in e) how do you expect Vera's demand for each food item to change due to changes in prices and budget? Represent graphically and interpret.

Chapter 10 – Intertemporal Choice

1) (EA Exam 08.01.2008). Assume that Concha's intertemporal consumption preferences are described by the following utility function:

$$U = \ln C_1 + \frac{\ln C_2}{1,075}$$

Income in periods 1 and 2 is, respectively, equal to m_1 and m_2. Concha can lend or borrow at a rate of 7.5%.

a) Write the expressions for the intertemporal budget constraint.

b) Graphically represent the solution for Concha's optimization problem.

c) Determine the levels of consumption chosen by Concha in periods 1 and 2.

d) Will Concha be a borrower or a lender in period 1? Determine the amount being lend or borrowed.

Chapter 12 – Uncertainty

1) (Exam of EE 22.07.08) The *right equivalent* of a lottery is the amount of money that a person needs to receive in order to achieve the same welfare level that would obtain with the lottery. Consider a person, whose von Neumann-Morgenstern utility function on a lottery, where when event 1 happens x money is received, and when event 1 does not happen the amount of money received is y, is the following:

$$U(x, y, \pi) = \pi\sqrt{x} + (1-\pi)\sqrt{y}$$

π is the probability of x to occur. Determine this lottery right equivalent, as function of x, *y* e π.

Chapter 15 – Market Demand

1) (Exam of EA 20.01.2011) In Springfield, the price elasticity of demand for the coffee sold in Apu's store is -1.24. Suppose Apu raises the price of coffee.

   a) Coffee consumed will also rise; True or False? Justify.

   b) Apu's revenues will also rise; True or False? Justify.

   Sometime latter, the estimated price elasticity of demand for Apu's coffee was -0.85.

   c) Speculate on what could have happened in Springfields's market for coffee for this to happen.

   d) In this case Apu's revenues will also rise. True or False? Justify and briefly interpret comparing to question b).

2) (Exam of EA 12.01.2005) Suppose the demand curve for a football game from Associação Desportiva do Monte da Amora (ADMA) is given by $D(p) = 2000 - 100\,p$, where *D(p)* is the demanded amount for a *p* price. The ADMA stadium has the capacity for 2000 spectators capacity. ADMA executives must set the price for game tickets.

   a) Which price generates maximum revenue? What amount of tickets is sold for that price?

   b) For the amount found previously, determine the marginal revenue and the price elasticity of demand.

   c) Following a series of winning games, fans' interest in the club increases, and the demand curve moves to $D(p) = 20000 - 100\,p$. Determine, in this situation, the price that brings maximum revenue and the amount of tickets sold for that price.

3) (Exam de EA 13.09.2005) Suppose the demand curve for a football game of Concha Mar F.C. (CMFC) is given by $D(p) = 20000 - 100\,p$, where *D(p)* is the amount demanded for a *p* price (expressed in euros). ADMA's stadium has a 2 000

spectators capacity, but member Romário Abraão, a generous patron of the club, made himself available to finance a stadium as big as the club's wish, upon a sole condition: the club needs to guarantee that the stadium is full in every match. If the club chooses to maximize its revenue of ticket sale, what stadium size does it choose?

4) (Exam EA 13.01.2006). The demand market for lemons is given by $y = 80€ - 4P$ (where the amount of lemons is given non-dimensionally). At what price is the total expense in lemons maximized? For that price, calculate the price elasticity of demand.

5) (Exam of EA 25.01.2006). Consider the demand function $y = 10 - 2P$.

   a) Represent the function and state the ranges where it is elastic, inelastic and unitary.

   b) Identify the point that corresponds to maximum total expenditure (that is, the maximum of total revenue for sellers of this good).

6) (Exam of EA 25.05.2006). Pedro and Carlos are brothers with identical musical preferences. Determine aggregate demand function of both, knowing that individual demand of CD's can be expressed by function $p/€ = 15 - x_i$ (with i=1,2, and x_i is non-dimensional, for it is the CD's number).

7) (Exam of EA 08.01.2008). Determine the market demand function of good x given the following individual demand functions:

$$\begin{aligned} x_i &= 10 - 0{,}1\left(p/€\right) & i &= 1,\ldots,10 \\ p/€ &= 30 - 2x_j & j &= 1,\ldots,5 \\ x_k &= 25 - 3{,}06\left(p/€\right) & k &= 1,\ldots,25 \end{aligned}$$

8) (Exam of EA 12.01.2007). Seventh Grade S. Pedro do Brilho class comprises 15 students that choose the same way when consuming Bobicaus.

   a) If individual demand is $y_i = 40 - 2\left(p/€\right)$, where p is the price of Bobicaus price, determine the class's demand function.

   b) Is Bobicaus' class demand elastic, inelastic or unitary elastic?

   c) For a monopolistic Bobicaus selling store in this school, at which price is the revenue maximized?

9) (Exam of EA 01.02.2007). Assuming that Portuguese consumption can be divided in two goods: veal meat (with consumption amount, in kg/year, given by v, and price, in €/kg, given by p); a generalized consumption good, that includes all remaining goods consumed in Portugal (with the amount spent in this good, in €/year, given by C). Suppose the utility function for each Portuguese person is given by $U = \left(v/\mathrm{kg}\right)^{1/100}\left(C/€\right)^{99/100}$. The per capita Portuguese income per unit time (e.g. €/year), is given by m.

   a) Determine for each Portuguese person the optimal consumption of veal meat and that of the generalized consumption good.

b) Assuming there are 10 million Portuguese people and that their income is 5.000 €/year, calculate the demand function of veal meat for Portugal. Discuss the plausibility of this function.

10) (Exam of EA 24.01.2009). Maria and António are the sole consumers of Bilaranjus soda, in a small village near Monfortinho. Their demand curves are given, respectively, by $p_1 = 12 - 0,5x_1$ and $p_2 = 10 - 0,5x_2$. What is the market demand curve for soda in the village?

11) (Env. Econ. Exam, 18.01.2014) Consider the demand functions for *x* and *y* obtained from the Cobb-Douglas utility function, $u(x_1, x_2) = kx_1^a x_2^b$, with the budget constraint $p_1 x_1 + p_2 x_2 = m$:

$$x_1 = \frac{a}{a+b}\frac{m}{p_1},\ x_2 = \frac{b}{a+b}\frac{m}{p_2}$$

a) Calculate the price demand elasticities for each of these demand functions.

b) What is the price for each good that maximises expenditure?

12) Consider two consumers *i*=1 and *i*=2, with demand curves (for positive quantities) given by $y = a - b_i p$, with $b_1 > b_2$.

a) Calculate the aggregate (market) demand curve.

b) Represent, on the same graph, the two individual demand curves and the aggregate demand curve.

c) Calculate the price elasticity of demand of the aggregate demand curve.

d) Calculate the price for which expenditure is maximum. Interpret.

Suppose the supply curve for this good is given by $MPC = b$.

e) Represent graphically the consumer surpluses of each consumer at the market equilibrium.

13) (EA Exam 26.01.2017) Consider the market for good *y* where a set of consumers has a demand function given by $y = A$, where *A* is a positive constant.

a) Calculate the price which maximises revenue for a monopolist selling in this market. Interpret.

Suppose now that there is a second set of consumers in this market that has an inverse demand function given by $p = ay + b$ for $y < b/a$, $a < 0$.

b) Calculate the aggregate demand in this market, considering both sets of consumers; represent on the same graph the two disaggregate demand functions and the aggregate demand function.

14) (EARN Exam 04.02.2020) A consulting company is conducting a market analysis for a new video game, sold in the Netherlands, called Chronic. Their statistics team has estimated the following demand curve for copies of Chronic sold to consumers with ages below 15 years old (≤ 15):

$$D(\leq 15) = 15000 - 300 \cdot p$$

a) Determine the marginal revenue and the price elasticity of demand.

b) What is the relationship between marginal revenue and price elasticity of demand? Show mathematically.

c) The consulting company has advised Panama Red – the makers of Chronic – to sell it at a price which maximized total revenue. What is that price? How many copies of the game are sold?
d) Sketch the demand curve and revenue on appropriate graphs, and identify the ranges in which the demand for Chronic is elastic, inelastic, and unitary, in the Netherlands. What happens to revenue when price is raised in each case? Justify.
e) Suppose now that the demand curve for video games sold to consumers above 15 years old (> 15) is given by

$$D(> 15) = 7500 - 100 \cdot p$$

i. Sketch, on separate graphs, the individual demand curves, and the market demand curve. Express the market demand curve mathematically.
ii. Assuming that the supply curve for videogames is given by $S = 250 \cdot p$, sketch the consumer surpluses for each of the two age ranges, at the market equilibrium.

15) (EARN Exam 09.09.2020) Filipe Ribeiro has a passion for action sports. He has a collection of skateboards, and another collection of surfboards (a collection of surfboards is called a quiver, apparently). Filipe doesn't know which collection he likes the most, but if he could write his preferences regarding skateboards (k) and surfboards (f), it would look something like this:

$$U(k, f) = (k + a.f)^{1/4}, \text{ with } a > 1$$

At his local shop, the price Filipe pays for a skateboard is p_k, and the price he pays for a surfboard is p_f. The budget Filipe has to spend on only these items is given by m.

a) Will Filipe be more satisfied purchasing one more skateboard? Or one more surfboard? Show mathematically. Determine the marginal rate of substitution (MRS) between skateboards and surfboards and interpret your results.
b) Determine whether, according to Filipe's preferences, skateboards and surfboards are perfect complements, perfect substitutes, or none of these. Sketch the family of indifference curves for the utility function to justify your answer.
c) Write Filipe's budget constraint and sketch it in relation to the indifference curves, for all possible cases. Identify a corner solution graphically. (Draw as many graphs as you need).
d) Solve Filipe's optimization problem, writing the candidate interior solution(s) and corner solutions.

Where Filipe lives, there is only one shop that sells skateboards, and the demand market for this product is given by the demand curve:

$$k = 80 - 4 \cdot p_k$$

e) At what price is the total revenue of the shop maximized? For that price, calculate the price elasticity of demand.

Chapter 16 – Equilibrium

1) The demand for champagne bottles is $1{,}000{,}000 - 60{,}000(p/€)$, where p is the price of champagne bottles. The supply curve for champagne bottles is $40{,}000(p/€)$.
   a) What is the equilibrium price in this market? What is the equilibrium quantity of sold champagne bottles?
   b) Suppose government introduces a tax such that each wine producer must pay a €5 tax for each bottle he produces. What is the new equilibrium price? And the new equilibrium quantity?

2) Consider a firm whose marginal costs curve is as follows:

$$MC(y) = a(y - b)^2 + c$$

where *a*, *b* and *c* are positive constants.

   a) Determine its supply curve.

Consider a market where there are *n* firms with this marginal cost curve.

   b) Determine the sector (or industry) supply curve.

Consider that in this market the demand curve is given by $p = Ay + b$, with A negative and *b* positive.

   c) Determine the price elasticity of demand in this curve.
   d) Represent graphically the market equilibrium, the producers' surplus and the consumers' surplus.

3) King Kanuta rules a small tropical island, Coco Atoll, whose primary crop is coconuts. Considering that the price of coconuts is given by p (national currency being US dollars), then the demand for coconuts on Coco Atoll is given by $D(p) = \frac{1200}{\text{week}} - \left(\frac{100}{\text{week}}\right)\frac{p}{\$}$. The supply of coconuts by the island coconut growers is given by $S(p) = \left(\frac{100}{\text{week}}\right)\frac{p}{\$}$.
   a) What is the producers' marginal costs curve?
   b) What are the prices and quantities of coconuts sold in equilibrium?
   c) One day, King Kanuta decided to tax his subjects in order to collect coconuts for the Royal Larder. The King required that every subject who consumed a coconut would have to pay a coconut to the king as a tax. Determine for these conditions, in equilibrium, the price of coconuts, total amount of coconuts produced, and quantity of coconuts consumed by the King's subjects.
   d) King Kanuta's subjects resented paying tax over coconuts, and whispers of revolution spread through the palace. Worried by the hostile atmosphere, the king changed the coconuts tax. Now, the shopkeepers who sold the coconuts would be responsible for paying the tax. For every coconut sold to a consumer,

the shopkeeper would have to pay a coconut to the king. In these conditions, determine, in equilibrium, the price of coconuts, total amount of coconuts produced, and quantity of coconuts consumed by the King's subjects.

4) (EA Exam Special Season 2008.) Consider a market with demand curve $p = k\left(y/y_0\right)^{-a}$ and supply curve $p = k\left(y/y_0\right)^{b}$, where a, b, k and y_0 are positive constants.

   a) Determine the price elasticity of demand. Is demand elastic, inelastic or does it have unit elasticity?

   b) Graphically represent demand and supply curves, as well as market equilibrium.

   c) At the market equilibrium, calculate the consumer and producer surpluses.

   d) Interpret how the parameter a affects the results.

5) (EA Exam 02.02.2011) Assume there are two consumers in the market for comic books, Bart and Millhouse. Their demand functions, in number of comic books, are respectively given by, $Q_M^D(p) = 12 - p$, and $Q_B^D(p) = 15 - 2p$. The only comic bookstore in Springfield is owned by the Comic Book Guy. Moreover, consider that the Comic Book Guy's supply function is given by $Q_C^S(p) = ap$.

   a) How many comic books do Bart and Millhouse buy for each of the following prices $p = 14€/book$, $p = 9€/book$, and $p = 4€/book$?

   b) Write down the aggregate demand function. Graph the demand for comic books (draw individual demand curves and the aggregate demand curve).

   c) Calculate the market equilibrium (price and quantity) assuming that $a = 2$. Represent in a supply-demand graphic. How many comic books do Bart and Millhouse each buy?

   d) Assume that the Comic Book Guy throughout the years was able to find out the demand curves for Bart and Millhouse. Since he does not want Bart in his store, he needs to change his supply curve (change a) in order to prevent Bart from buying any comic books. What could be his new supply curve, and how can he do this?

6) (EA Exam 26.01.2015) Consider firms with the production function $f(z_1, z_2) = k z_1 z_2$, where z_1 and z_2 are the quantities used of production factors 1 and 2. Consider also that the unit costs for these production factors are respectively w_1 and w_2.

   a) For a given production level y, calculate the values z_1 and z_2 that allow it to be produced in the cheapest way. Calculate the cost curve and the marginal cost curve. Interpret the result.

Suppose now that the firm's production activities generate pollution, and that it pays a tax t per unit of production.

   b) Repeat the calculation in a), considering this extra cost. Interpret.

Consider now the consumers of this good and a generalised good x, with a Cobb-Douglas utility function $u(x, y) = cx^a y^b$, subject to the budget constraint $p_1 x + p_2 y = m$.

c) Calculate the optimal quantities x and y consumed.

Suppose the tax on the pollution generated by producing y is now paid by the consumer.

d) Write the budget constraint now faced by the consumer.

e) Calculate the optimal quantities x' and y' now consumed.

f) Show that the market equilibrium for good y reached by these producers and consumers is the same, regardless of whether the tax is paid by the producer or the consumer.

Chapter 17 – Technology

1) Consider the Cobb-Douglas production function, where K and L are production factors:

$$f(K,L) = AK^a L^b$$

a) Calculate the marginal product for each production factor.

b) Determine whether this function has constant, decreasing or increasing returns to scale.

c) Sketch the isoquants for this production function.

2) (Exam EA 12.01.2005). One firm has a production function $f(x,y) = \min\{2x, x+y\}$, and another has a production function $g(x,y) = x + \min\{x,y\}$.

a) Show graphically the isoquants for each of these production functions.

b) Determine for each function whether there are decreasing, constant or increasing returns to sale.

3) (EA Exam 13.09.2005). Consider the production function $f(x_1, x_2) = \sqrt{x_1} + x_2^2$.

a) Calculate the marginal products for each factor.

b) Has the function decreasing, constant, or increasing returns to scale?

4) (EA Exam 28.01.2005). Consider the production function $f(K,L) = aL/2 + b\sqrt{K}$

a) Determine whether it has decreasing, constant or increasing returns to scale.

b) Determine whether the marginal product of labour is decreasing, constant or increasing.

5) (EE Exam 23.01.2019) Consider the LINEX production function, proposed by Kummel to consider the contribution of energy to production:

$$y = y_0\, e \exp\left(a_0\left(2 - \frac{l+e}{k}\right) + a_0 c_0\left(\frac{l}{e} - 1\right)\right)$$

where e, l and k are the non-dimensional values of energy, labour and capital, respectively (i.e., all the values are normalised by a reference value) and y is GDP.

a) Does this function have decreasing, constant or increasing returns to scale? What is the economic meaning of this?

b) Calculate the marginal productivities (hint: to simplify the calculations, first take logarithms of both sides of the above equation and, if helpful, express the results as a function not only of *e*, *l* and *k,* but also of *y*).

c) Calculate the output elasticities (which are in theory equal to the cost shares), i.e.,

$$\frac{e}{y}\frac{\partial y}{\partial e}, \frac{l}{y}\frac{\partial y}{\partial l}, \frac{k}{y}\frac{\partial y}{\partial k}$$

d) (0.5) Calculate the sum of the three output elasticities.

e) (1.5) Interpret the results in b) c) and d).

6) (EE Exam 24.01.2009) Depict isoquant families associated to the following production functions (where all parameters are positive) and state whether they are perfect complements, perfect substitutes or something else:

a) $y = ax_1^2 + bx_2^2$,

b) $y = \min\{x_1/2, x_2\}$.

7) (EA Exam 17.01.2015) The Constant Elasticity of Substitution production function is given by the expression, where z_1 and z_2 are production factors and a and r are positive constants, with $a < 1$:

$f(z_1, z_2) = \left(az_1^r + (1-a)z_2^r\right)^{1/r}$

a) Does this function have decreasing, constant or increasing returns to scale?

b) Calculate the marginal productivity of z_1; discuss the effect of a on this result.

Chapter 18 – Profit Maximization

1) (Exam EA 28.01.2005). Mr. Castro discovered he can get a quantity W of wheat production per unit area if he does not use fertiliser. If he applies an amount *N* of nitrogen fertiliser per unit area, the marginal product of fertilisation is $k(1 - N/N^*)$, where *k* and N^* are positive constants.

a) If the wheat's selling price is *p* and the unit cost of fertiliser is *w*, what amount of fertilisation will maximise Mr. Castro's profit?

b) Write an expression that gives Mr. Castro's production per hectare, as a function of the amount of fertiliser he uses.

2) (Exam ENRE 12.01.2018) The output of an economy, which can be measured as GDP, Gross Domestic Product, is conventionally given as a function of two production factors, capital (e.g., machines), *K*, and labour, *L* (i.e., workers):

$$GDP = f(K, L)$$

The Cobb-Douglas function is a common choice for the production function *f*:

$$f(K, L) = AK^a L^b$$

where *A, a* and *b* are positive constants and $a + b = 1$.

a) Check that *f* has constant returns to scale; what do you think is the meaning of this property at the national level?

b) Calculate the marginal productivities for this production function. Interpret.

We can define the output elasticities for each production factor as:

$$\frac{K}{f}\frac{\partial f}{\partial K}, \qquad \frac{L}{f}\frac{\partial f}{\partial L}$$

c) Calculate the output elasticities for the Cobb-Douglas production function.

d) Are these elasticities elastic, inelastic or unit-elastic? Interpret.

e) Verify the Euler equation $f = MP_1K + MP_2L$ in this case (note that the equation is valid for all functions with constant returns to scale, which mathematically correspond to homogeneous degree 1 functions).

Since production factors are paid according to their marginal productivities, we have $r = MP_1$ and $w = MP_2$, where *r* is the return on capital (interest, dividends, rents) and *w* is wages.

f) Using the Euler equation considered in e), conclude that all GDP is paid to capital and labour.

3) (EE Exam 01.02.2019) Consider the Stone-Gerry production function, a variation of the Cobb-Douglas production function that considers the existence of a threshold factor requirement (i.e., x_0 and z_0, with $x>x_0>0$ and $z>z_0>0$) for each output (with *A*, α and β being positive quantities, $\alpha + \beta = 1$):

$$y = A(x - x_0)^{\alpha}(z - z_0)^{\beta}$$

a) Does this function have constant returns to scale? Interpret.

b) Sketch the family of isoquants (constant production lines) for this production function.

c) Calculate the marginal productivity for each production factor. Interpret.

d) Calculate the technical rate of substitution. Interpret.

Suppose a producer sells the good *y* at price *p* and pays the production factors *x* and *z* at prices w_1 and w_2, respectively.

e) Calculate the optimal amounts of *x* and *z*. Interpret.

Chapter 19 – Cost Minimization

1) (Exam EA 14.01.2017) Consider the Cobb-Douglas production function in the macroeconomic setting, $f(K, L) = AK^aL^b$, where *K* is capital, *L* is labour, *A*, *a* and *b* are positive constants and $a+b=1$. Suppose the cost of capital (the interest rate) is *r* and the cost of labour (the wage) is *w*.

a) Does this production function have decreasing, constant or increasing returns to scale?

b) For a given production level *y*, calculate the optimal amount of *K* and *L* used.

c) Interpret the effect of *A*, *r, w,* and *y* on these optimal amounts.

Chapter 20 – Cost Curves

1) A firm manufactures two identical products in two different plants. If marginal cost in the first plant is superior to marginal cost in second plant, how can the firm reduce costs, maintaining the same level of production?

2) (Exam EA 13.01.2006). Consider the following production function of a certain firm: $y = KL^2$, where K is the capital factor and L is the labour factor, both measured in non-dimensional amounts, as well as the production.

   a) What kind of return to scale does this firm present?

   b) Determine total cost, average cost and marginal cost expressions, considering the firm contracts capital and labour factors w_1 e w_2, respectively.

3) (Exam EA 12.01.2007). Consider a firm with the average cost function $AC = 9y + 3 + 9/y$. Determine this firm's cost curve.

4) (Exam EA 08.01.2008). Consider a firm whose production function is $f(K,L) = a\sqrt{KL}$, where K is the amount of capital used, L is the amount of labour used and a is a positive constant.

   a) Determine whether the returns to scale of this function are decreasing, constants or positive.

   b) Sketch the isoquants of these functions.

   c) If the costs of production factors K and L are respectively w_1 e w_2, both positive, determine the minimum production cost of amount y, and the corresponding amounts of production factors used. Represent graphically the solution.

   d) Determine the fractions of production cost spent with each production factor. Express verbally the result.

   e) Interpret the effect of w_1, w_2 and y on the cost and on the amounts of production factors.

5) (Exam EE 11.07.2008). A university student is considering for summer whether he will devote himself to his family business, where he is paid a unitary value of w_1, or whether he should launch a lawn cutting activity. For this, he will need to buy fuel at a unit cost of w_2 and rent a lawn mower. It could be a small lawn mower, which cuts one unit area per unit time, with fuel consumption equal to 1/3 per unit time and a unit cost of w_3. It could also be a big lawn mower, which cuts three area units per time unit, with a fuel consumption of one per time unit and unit cost of w_3. Consider z_1 is the labour time, z_2 is the fuel consumption and z_3 and z_4 are time spent using respectively the small lawn-cutter and the big lawn-cutter. The production functions of small lawn-cutter and big lawn-cutter are respectively (considering all amounts non-dimensional):

$$y = min\{z_1, 3z_2, z_3\}$$

$$y = 3\,min\{z_1, z_2, z_4\}$$

   a) Obtain the cost curves for this activity, for both the small lawn-cutter and the big lawn-cutter use.

   b) Determine the conditions where using the small lawn-cutter is cheaper than using the big lawn-cutter. Interpret the result, particularly, explaining the reason why result is independent from fuel's price.

   c) What should be the price charged per unit area of lawn cutting, so that for the student it compensates leaving the family business and dedicating himself to this activity.

6) (Exam of EA 07.09.2009) Consider the following cost function:

$$C(y) = 4y^2 + 16$$

   a) Calculate the average cost curve.

   b) Calculate the marginal cost curve.

   c) Determine the level of output at which the average production cost is minimum.

   d) Determine the average variable cost curve.

   e) For which level of production does average variable cost equal marginal cost?

7) (EA Exam 12.02.2009) Assume that the retail price of potatoes is equal to its average cost of production. Explain how farmers' profits will change with changes in the price of potatoes, while the average cost of production stays constant.

Chapter 21 – Firm Supply

1) A firm has a cost function $c(y) = 10\text{€}\,(y/\text{kg})^2 + 1000\text{€}$.
   a) What is its supply curve?
   b) At what level of production does the firm minimize its average cost?

2) (EA Exam 01.02.2017) Assume that a firm's cost function is given by $C(y) = 10 - y + y^2$

   a) What are the fixed costs of this firm?

   b) What are the variable costs?

   c) What are the average variable costs?

   d) Calculate and represent graphically the short-run supply function for this firm. Interpret.

3) A handicrafts store has a production function $f(x_1, x_2) = \left(\min\{x_1, 2x_2\}\right)^{1/2}$, where x_1 is the amount of plastic used, x_2 is the amount of labour used and $f(x_1, x_2)$ is the number of parts produced. Let w_1 be the price per unit of plastic and w_2 the wage per unit of labour.
   a) For a production level y, determine the minimum value the store spends, i.e., determine its cost function $c(w_1, w_2, y)$.

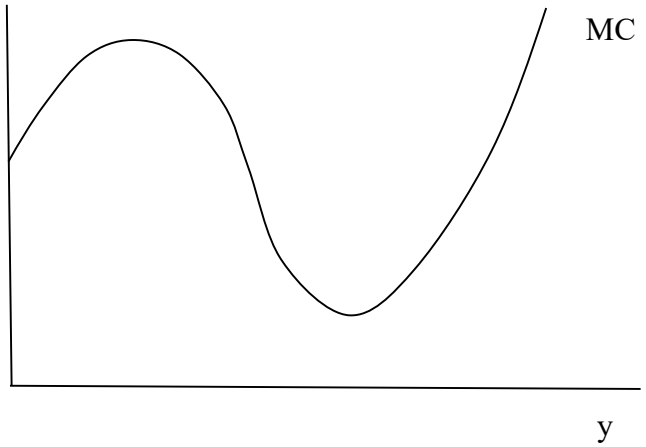
   b) Calculate marginal cost, $MC(y)$ the supply curve, $S(p)$, and the average cost, $AC(y)$.

4) (Exam 25.01.2006). A firm's cost function is $C/\text{€} = y^3 - 4y^2 + 8y + 10$, where y is the production level, in non-dimensional quantities.
   a) Calculate the FC (fixed cost), VC (variable cost), MC (marginal cost), AC (average cost) and AVC (average variable cost) curves.
   b) Represent graphically MC, AC and AVC.
   c) Determine this firm's short run and long run supply curves.
   d) If selling price is p, what level of production will maximize the firm's profits?

5) (Exam EA 25.05.2006). Consider the production function $f(x_1, x_2) = (2x_1 + x_2)^{1/2}$, where x_1 and x_2 are the amounts used for each production factor. Consider prices of each production factor are respectively w_1 e w_2.

a) Determine the minimum cost of production for a y level of production.
b) Determine the marginal cost of production.
c) Determine the supply curve.

6) (EA Exam 08.01.2008). Consider a firm with non-dimensional production cost is $C(y) = y^3 - 6y^2 + 15y + 100$, where y is the level of production, expressed non-dimensionally.
   a) Calculate and represent graphically the marginal cost and average variable cost curves.
   b) Calculate and represent graphically the short-term supply curve.
7) (ENRE Exam 31.01.2018) Suppose a firm produces a certain good in quantity y, using a production factor in quantity x, with unit cost w_1, with production function $f(x) = 2bx^{1/2}$, and sells this good at price p. Suppose also that the production by this firm generates pollution π, proportional to the amount of production factor x used, i.e., $\pi = \varepsilon . x$.
   a) Give a real-world example of a situation where pollution generated is like this.
   b) Suppose that the firm has the option of carrying pollution abatement, i.e., pollution control, in an amount a, with pollution abatement cost equal to $w_2 a^2$. Give a real-world example of a situation like this.
   c) Suppose also that the firm pays an environmental tax t on the pollution it does not abate. The profit function for this firm is thus
   $$2pbx^{1/2} - w_1 x - w_2 a^2 - t(\varepsilon . x - a)$$
   Calculate the optimal level of use of the production factor x and of the abatement level a; calculate the optimal production level y.
   d) Discuss the effect of the value of the environmental tax on the optimal decision of the firm, namely considering (i) how increasing the tax changes the values of x and a, and (ii) the maximum plausible value for t.
   e) Calculate the minimum cost for obtaining a supply level equal to y. What are then the values of x and a?
   f) What is the marginal cost, the variable cost and the fixed cost for this firm? Interpret.
   g) What is the short run supply curve of this firm? Relate this answer to your answer to c).
8) (EA Exam 01.02.2007). Given the marginal cost curve (MC) shown below, represent graphically the average variable cost curve (AVC) and the short-term supply curve. Justify the latter.



Chapter 31 – Welfare

1) (EE Exam 11.07.2008). One possible method of determining a social preference relation is the *Borda Count*, also known as rank-order voting. Each voter is asked to rank all the alternatives. If there are ten alternatives, you give your first choice a 1, your second choice a 2, and so on. The voters' scores for each alternative are then added over all voters. The total score for an alternative is called its Borda count. For any two alternatives, x and y, if the Borda count of x is smaller than or the same as the Borda count for y, then x is "socially at least as good as" y. Suppose that there are a finite number of alternatives to choose from and that every individual has complete, reflexive, and transitive preferences. For the time being, let us also suppose that individuals are never indifferent between any two different alternatives but always prefer one to the other.
   a) Is this social preference relation complete? Is it reflexive? Is it transitive?
   b) If everyone prefers x to y, will the Borda count rank x as socially preferred to y?
   c) Does the social preference relation defined by the Borda count have the property that social preferences between x and y depend only on how people rank x versus y and not on how they rank other alternatives?

Ethics

1) According to utilitarianism:
   a) we should all follow rules, if it improves the sum of all individuals' well-being;
   b) we should all follow rules, for it protects the weakest from the strongest
   c) we should all follow rules, if, when someone stops following rules while everyone else is obeying, his well-being gain is inferior to the sum of well-being losses of everyone else.
   d) a) and c) are correct.
2) (Exam of EE 11.07.08) Discuss, from the viewpoint of the ethical theories of Kant, Rawls and utilitarianism whether:
   a) a market for carbon dioxide emissions permits should exist;
   b) the initial allocation of these permits should be free or auctioned.
3) Given a specific situation in our society, Rawls' ethic theory says we should remove resources from the poorer to the richer if
   a) the poorer becomes happier in consequence;
   b) the poorer are less powerful than the richer;
   c) the happiness lost by the poorer is inferior to the happiness gained by the richer;
   d) everyone agrees one should do it.
4) According to Kant's Categorical Imperative:
   a) I shall not lie, because it is something I dislike;
   b) I shall not lie, because I would not wish everyone else did it;
   c) I shall not lie, because it makes some people less happy;

d) I shall lie, if there is positive balance between happiness and unhappiness it causes to everyone;

5) Utilitarianism accepts removing resources from the poorer to the richer if
   a) the poor are less powerful than the rich;
   b) the happiness the poor lose is inferior to the happiness the rich gain;
   c) everyone agrees one should do it;
   d) the poor become happier in consequence.
6) Rawls' ethic theory was developed for human societies. Discuss how it could be applied to a naturalist or ecocentric context (maximum 250 words).
7) (EA Exam 12.01.2005). Consider Brundtland's definition of sustainable development: "A development that allows suppressing actual generations' needs without compromising future generations' capacity of suppressing theirs".
   a) Discuss this definition's relation to Kant's ethic theories, Rawls' and utilitarianism (maximum 150 words).
8) (EA Exam 28.01.2005). Discuss what recommendations would Kant's ethical theories, Rawls' and utilitarianism give to someone hesitating between abstaining in the next legislative elections or voting (no matter the vote's outcome) (maximum 150 words)
9) (EA Exam 13.09.2005). According to utilitarianism a social decision rule is ethical if (discuss the correct option, maximum length approximately 100 words)
   a) it benefits people who hold power;
   b) the sum of utility variations which that rule causes in all members of society, is superior to that caused by any other social rule;
   c) the rule is judged as intrinsically right, based on community shared social values;
   d) the costs exceed the benefits for the whole society, where the costs are being supported by a small number of individuals and the benefits are received by the majority of society's individuals.
10) (EE Exam 24.01.2009) Consider the simplified version of the World that divides it between "North" (rich countries) and "South" (poor countries). Consider greenhouse gas emissions, which are mainly emitted by the "North", but whose effects will affect both "North" and "South". Discuss, based on Kant's ethic theories, Rawls' and utilitarianism, how should the effort of reducing greenhouse effect gases be distributed between "North" and "South" (maximum length of approximately 300 words).
11) (EA Exam 13.01.2006). Consider the following "Calvin & Hobbes" cartoon.

Calvin: Some people are pragmatic, they accept things as they are and make the best possible choice.
Calvin: Some people are idealists, they defend principles and reject compromises.
Calvin: And some people act only upon their whims.
Hobbes: What kind are you, I wonder.
Calvin: I pragmatically turn whims into principles!

Consider the way Calvin structures the three approaches he presented. Discuss how such a structure could be done for the three ethical theories you've learnt: Kant, utilitarianism, Rawls (maximum approx. 300 words).

12) (EA Exam 25.05.2006). Kant proposed an ethical theory based on (discuss the correct option; maximum length approximately 100 words)
   a) the assumption of an open society, including values, cultures and diverse attitudes towards civic virtue;
   b) the duty or moral obligation of choosing an intrinsically correct action, whatever the desirability of the foreseen outcome of that action might be;
   c) the desirability of the foreseen outcome of the action;
   d) the use of the Earth's resources to satisfy human needs.
13) (EA Exam 12.01.2007). Consider the possibility of establishing in Portugal a market for kidney donation. Analyse such a possibility in the light of Kant's, utilitarianism's and Rawls's ethical theories (maximum circa 250 words).
14) (EA 12.01.2007). Discuss which answer would Kant's, utilitarianism and Rawls ethical theories recommend to the referendum question on voluntary interruption of pregnancy.
15) (Exam of EE 11.07.08) Discuss, from the viewpoint of the ethical theories of Kant, Rawls and utilitarianism whether:
   a) a market for carbon dioxide emissions permits should exist;
   b) the initial allocation of these permits should be free or auctioned.
16) (Exam of EE 22.07.08). Discuss, according to the ethical theories of Kant, Rawls and utilitarianism, the validity of the definitions of sustainable development given by strong sustainability and weak sustainability.

17) (Exam of EA 12.01.2009). Discuss, from the viewpoint of the ethical theories of Kant, Rawls and utilitarianism, the Business and Biodiversity program, a brief description of which is provided below:

> The main objective of this initiative of the European Union is to increase the relationship between business and biodiversity, enabling it to give a significant contribution to biodiversity protection (...).
> The initiative seeks to promote, through long-term voluntary agreements, the introduction of biodiversity in firms' strategies and policies.
> (...) The methodology is based on an assessment of the effect of business on biodiversity, on which is structured an action plan for this, resulting in concrete projects with positive impacts on biodiversity. (...) Biodiversity as a public good requires the involvement of the authorities of the Member States and the Commission as promoters of this initiative, giving public recognition to participating firms. (source: http://portal.icn.pt).

18) (EA Exam 20.01.2011). Analyse, according to the ethical theories of Kant, Utilitarianism, Rawls and Libertarianism, whether smoking should be forbidden in public outdoor spaces (maximum length of approximately 300 words).

19) (EA Exam 02.02.2011). Analyse, according to the ethical theories of Kant, Utilitarianism, Rawls and Libertarianism, whether farmers should pay for the water they consume (maximum length of approximately 300 words).

20) (EA Exam 01.02. 2017) Give an example of a real-world ethical decision where utilitarians, libertarians, Kantians and Rawlsians would all agree on the correct course of action.

21) (ENRE Exam 12.01.2018) Let us explore what may be called a "Rawlsian" view of society. Consider, for simplicity, a society with just two individuals, where the welfare of society is given by $u(x_1, x_2) = \min(ax_1, bx_2)$, with x_1 the amount of resources allocated to individual 1 and x_2 the amount of resources allocated to individual 2. There is an overall resource constraint given by $x_1 + x_2 = X$, where X, a constant, is the total amount of available resources. Note that the welfare function can be represented as a function with branches:

$$u(x_1, x_2) = \begin{cases} bx_2 & \text{for} \quad ax_1 \geq bx_2 \\ ax_1 & \text{for} \quad ax_1 < bx_2 \end{cases}.$$

a) Sketch the family of indifference curves for this utility function.

b) Represent graphically the optimal solution. Interpret.

c) Formulate the Lagrangian and calculate the candidate interior solution(s). If necessary, calculate the corner solution(s) and find the optimal one. Interpret, including a discussion of the adequate values for a and b.

d) Can utilitarianism, libertarianism and Kant also be expressed in this way? If yes, how? If not, why?

22) (ENRE Exam, 31.01.2018) Consider the following simplified version of the Human Development Index, used by the United Nations to rank nations regarding "human development" (where x is an index of life expectancy and y is an index of economic output):

$$HDI^{\text{simpl}} = kx^{1/2}y^{1/2}$$

a) Sketch the family of indifference curves for this index.

b) Suppose a country's government decides to base its policies on increasing the HDI of the country. Discuss the positions on this that followers of the ethical theories of utilitarianism, libertarianism, Kant and Rawls would have.

23) (EARN Exam 17.01.2020) Suppose you would like to purchase a mountain bike. Consider the information presented in the following list.

- Goal: purchase a mountain bike.
- Criteria: price (the cheapest, the better), gear action (the more gears, the better), durability (the higher, the better).
- Options: three bikes you are undecided about: bike A, bike B and bike C.

The following impact matrix has been developed for these criteria:

Impact matrix			
Bike	Price	Gear action	Durability
A	100 €	1 gear	1
B	300 €	3 gears	3
C	700 €	18 gears	10

Weights of the criteria:

Criteria	Weight
Price	10
Gear	6
Durability	3

a) Should you consider other stakeholders in this decision-process? If so, who?

b) Based on the technique "Direct analysis of the impact matrix", and on the impact matrix presented above, i) can you select a best option? ii) if price was not a problem, could you select a best option? justify your answers

c) Based on the Linear Additive Model, which option is the best?

24) (EARN Exam 04.02.2020) "Different people favor different principles, reflecting their various interests, moral and religious beliefs, and social positions. Some people are rich and some are poor; some are powerful and well connected; others, less so. Some are members of racial, ethnic, or religious minorities, others not." (M. Sandel, *Justice*, Chap. 6, p. 141)

a) What would be the perspective of the four ethical theories studied in this course (utilitarianism, libertarianism, Kant, Rawls) on why it is relevant to ensure multiple perspectives are included in the process of evaluating/comparing different alternatives?

b) In a multicriteria decision process, how can you include these multiple perspectives?

25) (EARN Exam 09.09.2020) The Human Development Index (HDI) is an indicator created in 1990 by the United Nations Development Program to measure social progress. This indicator can be seen as a multicriteria application, with the following approach:

Criteria / indicators	**Units for scoring**	**Normal-isation**	**Weights**	**Aggregation procedure**
Life expectancy at birth	year	0 – 1 scale (a)	1/3	Linear additive sum
Expected years of schooling (for children of school entering age)	year	0 – 1 scale (b)	1/6	
Mean years of schooling (for adults aged 25)	year	0 – 1 scale (c)	1/6	
(natural logarithm of) Gross National Income per capita	ln €/capita	0 – 1 scale (d)	1/3	

(a) minimum 20 years of age, maximum 85 years of age; (b) minimum 0, maximum 18; (c) minimum 0, maximum 15; (d) minimum 100€/capita, maximum 75000€/capita.

a) Choose and justify the answer from the list below (1, 2, 3, 4, 5 or 6) that best completes the following sentence:

The Human Development Index follows a procedure coherent with...

1. Direct analysis of the performance matrix
2. Multi-attribute utility theory
3. Linear additive model
4. Analytical Hierarchy Process
5. Ouranking methods
6. Qualitative MCA

In the table below you find the Human Development Index (for 2017) and its four components for 6 countries.

	HDI	**Life expectancy at birth**		**Expected years of schooling**		**Mean years of schooling**		**Gross national income per capita**	
Units:	**value**	**years**	**(0-1)**	**years**	**(0-1)**	**years**	**(0-1)**	**ln ($/capita)**	**(0-1)**
Norway	0.953	82.3	0.958	17.9	0.994	12.6	0.840	11.13	0.985
Switzerland	0.944	83.5	0.977	16.2	0.900	13.4	0.893	10.96	0.960
Australia	0.939	83.1	0.971	22.9	1.272	12.9	0.860	10.68	0.918

Ireland	0.938	81.6	0.948	19.6	1.089	12.5	0.833	10.89	0.950
Germany	0.936	81.2	0.942	17	0.944	14.1	0.940	10.74	0.927
Iceland	0.935	82.9	0.968	19.3	1.072	12.4	0.827	10.73	0.926

According to the table, the ranking of the countries in terms of the HDI were:

Norway > Switzerland > Australia > Ireland > Germany > Iceland

b) If you were to use the MCA method "Direct Analysis of the Performance Matrix", which would be the ranking of the countries, if any? Provide a very brief justification of the reasons why you defined such ranking.

The way the Human Development Index is calculated can raise a few concerns that we have discussed in the class, such as:

- Given that the values are aggregated for a country, how to guarantee that minority groups situation are not loss/dissolved by the majority situation;
- Assume that health, education and income can be measured and compared in a single scale, by using quantitative criteria only (quality of health and educational systems are not evaluated), using weights and aggregating values together, and not reflecting on potential uncertainty in the data.

c) In the class we have explored a few approaches that could be used to address these aspects. Select from the list below between one to three techniques that you consider that address the problems above and justify your answer:

1. Multi-attribute utility theory
2. Linear additive model
3. Multicriteria mapping
4. Direct analysis of the performance matrix
5. Social multicriteria-evaluation
6. Outranking methods

Public Goods, Externalities and Values

In multiple choice questions, choose, justifying, the most correct.

1) In a country where the police are ineffective, an ice cream is a good which is:
   a) rival and excludable;
   b) non rival and excludable;
   c) rival and non-excludable;
   d) non-rival and non-excludable.
2) The value of biodiversity existence is a good which is
   a) rival and excludable;
   b) non rival and excludable;
   c) rival and non-excludable;
   d) non-rival and non-excludable.
3) A firm that pollutes water causes a negative externality because:

a) it damages a good that shouldn't be damaged;

b) it causes death to various organisms;

c) it affects people and firms, without paying the damages;

d) none of the above.

Pollution Economics

1) Consider the market for cookies. Pastrymen produce cookies with a marginal cost of 0.05 €/cookie. However, the production of each cookie generates external benefits for bypassers since it produces a pleasant smell, being such benefit equal to 0.02 €/cookie.

   a) Draw a plausible demand curve for the cookies, the marginal private cost curve, and the marginal social benefit curve.

   b) In the competitive market equilibrium, identify all contributions to net social benefit from the production of cookies.

   c) Given that there is a positive externality, is over- or under-production of cookies happening?

2) (EA Exam 13.01.2006). The graphic below represents the market of a good whose production generates external costs (MSC – Marginal Social Costs; MPC – Marginal Private Costs, or supply curve; MSB – Marginal Social Benefits, or demand curve). Use the graphic and the marked points to answer the following questions:

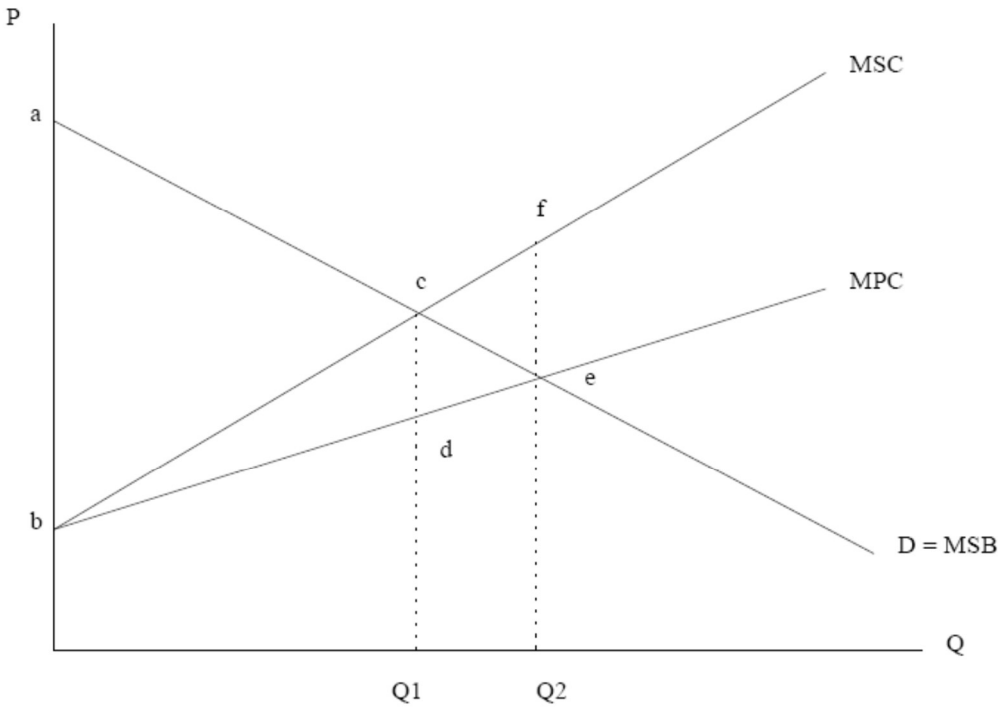
   a) Identify the market equilibrium.

   b) Identify the social optimum.

   c) Moving from Q_2 to Q_1, what does area cde represent?

   d) Moving from Q_2 to Q_1, what does area cdef represent?

   e) Which area represents the net gain to society which results from moving from the market equilibrium to the social optimum?



3) (EA Exam 25.01.2006). Consider two thermoelectric plants on coal with marginal depollution costs given by the following expressions:
   a) $MC_1 = 50 - 0.05e_1$
   b) $MC_2 = 50 - 0.2e_2$

   where e_1 and e_2 are each plant's emissions (note that these expressions implicitly indicate polluting emissions on absence of depollution, respectively equal to 1000 and 250).
   External marginal costs of these emissions, that include damages caused by acid rains and mercury contamination, are given by:
   MEC = 0.15 and
   where $e = e_1 + e_2$ is the total amount of emissions from the two plants.

   c) Show that marginal costs of pollution control for the set of both plants are given by MC = 50 − 0.04 *e*. (Hint: use the same approach applied to obtain demand market curve from de demand curves of each consumer).
   d) If there's no government intervention what is the level of emissions for each plant?
   e) What is the optimal total emissions level? Show graphically the solution.
   f) Given the former optimal level, what is the optimal distribution of depolluting effort between both plants?

4) (EA Exam 12.01.2007; SOLVED). Consider a situation where a firm has marginal costs given by $MPC = 10 + 0.5Q$, is in a market for which the demand curve is given by $MPB = 30 - 0.5Q$, but whose activity produces externalities, given by an external marginal costs function equal to $MEC = 10$. Q is the produced quantity.
   a) Calculate the market equilibrium. For this point calculate the producers' surplus, the consumers' surplus and the external cost.

b) Calculate the social optimum.

c) Calculate the Pigou tax value that allows bringing the market equilibrium to the social optimum. Calculate the variation in consumers' surplus, in producers' surplus, and in the externality. Based on these results, discuss the benefits of introducing the Pigou tax.

5) (EA Exam 01.02.2007). The "permanent sown biodiverse pastures rich in legumes" system is being implemented in Portugal. This system is simultaneously more productive, allowing farmers to have more animals and therefore more meat production to market sale, and at the same time leads to increases in organic matter on soil, that allow to fixate carbon and thus reduce the amount of carbon dioxide in atmosphere.

Suppose that:

- the marginal private cost of these pastures is given by $MPC = 150€/\text{ha.year}$;
- these pastures are only used to feed calves;
- these pastures allow production of 1 calf per hectare/year, corresponding to a production of 150 kg of calf meat per hectare/year;
- the inverse demand function for calf meat in Portugal is given by $p = 2€.\text{Mton}^{-1}\left(3 - \frac{y}{(\text{Mton/year})}\right)$, where y is annual meat consumption;
- these pastures allow a carbon sequestration equal to 5 ton CO_2/year.ha; assuming a valuation of 10 €/ton CO_2, we then have a marginal external benefit of pastures equal to 50 €/ha.year.

a) Represent graphically, making the pastures area, A, your abscissa, the marginal private cost, marginal private benefit and marginal external benefit curves.

b) Calculate the market equilibrium. Determine producer's and consumer's surplus and external benefit on this point.

c) Calculate the social optimum. Determine the producer and consumer surplus and external benefit at this point.

d) Calculate the Pigou subsidy, paid to farmers, that leads the market equilibrium to the social optimum. Determine the social gain from introducing the Pigou subsidy.

6) Supose you own a house by the river, where you enjoy swimming and fishing every day after work. Suppose also that there is a plant upstream of your house, that emits liquid effluents to the river. The plant's product has the demand curve $P = 300€ - (6€)Q$, where P is the price and Q is the (non-dimensional) amount and the marginal cost curve for the plant is $P = 30€ + (1,5€)Q$.

a) Assuming it is a perfect competition market with no state intervention, what is the level of production chosen by the plant, and what is the price? What are the values for consumer surplus, producer surplus and total surplus? Before obtaining the values, show them graphically.

b) Suppose the effluent loaded in the river is giving you an allergy, and the fishes you catch have more than two eyes. A very well-paid consulting firm estimates the external marginal costs of these impacts to be given by:

$$MEC = (1{,}5€)Q\,.$$

What is the optimal social level of production for the plant? What is the price at that level? Present both graphically.

c) What are the consumer and producer surplus values? Represent graphically.

d) Does going from the perfect competition solution to the social optimum increase the sum of consumer and producer surpluses? Represent graphically.

e) What are the total external costs for both the perfect competition solution and the social optimum? Present them graphically. Why isn't total external cost zero in the social optimum?

f) Represent graphically the area representing the net gain of going to the social optimum.

g) Suppose the plant has the legal right to pollute. Show how a bargain beteween you and the plant can lead to a optimal solution for both. Show graphically your maximum willingness to pay and your minimum willingness to receive from the plant.

h) Indicate a value for the total payment (for passing from the free market solution to optimal social solution) acceptable for both.

i) Suppose, alternatively, that you have the right to clean water, and that the plant has to negociate over a payment for you to accept pollution, in order to increase production to a optimal social level. Show how bargaining could lead to a optimal solution for both of you.

j) Explain how the existence of high transaction costs would change the solution above. Which could be those transaction costs?

7) (EA Exam 25.05.2006). Kym Anderson developed a simple model to analyse the effect of opening a small economy with external costs to the world market. The marginal private benefits, *MB*, the marginal private costs, *MC*, and the marginal social costs (private plus external), *MSC*, are given by $MB/€ = 5 - q/2$, $MC/€ = 1 + q/2$, $MSC/€ = 1 + 5q/6$, where q is the produced amount of a given good (measured non-dimensionally).

a) What is the price and produced quantity, without government intervention?

b) What should be the produced quantity, at the social optimum?

c) What is the value of a constant tax on producers, a Pigou tax, which would take the economy to the social optimum?

d) Show graphically the social welfare gain (as compared to the market equilibrium without government intervention) associated to the application of that tax. Indicate graphically the terms whose algebraic sum leads to this gain.

Now suppose the country starts to transact this good in the world market. This country's economy is small in comparison to world market, therefore any economic decision taken by it does not change the price of the good, that will equal its value in the world market, P = 2.

e) In the absence of government intervention, how much would be imported or exported?

f) In the presence of the above calculated tax, how much would be imported or exported (note that the tax only affects production in the country)?

8) (Exam of EA 09.02.2008). Consider a market with a negative externality, with $MPC = by$, $MEC = ay^2$, $MPB = MSB = c$, where a, b and c are positive constants and y is the produced quantity.

a) Represent the market equilibrium in perfect competition, indicating graphically the producer and the consumer surpluses and the externality.

b) Represent the social optimum, indicating graphically the producer and consumer surpluses and the externality.

c) Represent graphically the net social gain of going from the market equilibrium to the social optimum.

d) Represent graphically the Pigou tax.

e) Represent the market equilibrium when the Pigou tax is applied to the producer, indicating graphically the producer surplus, the consumer surplus and the total amount of tax paid.

9) (Exam 08.01.2008). Consider a market with a negative externality, with $MPC = by$, $MEC = ay^2$, $MPB = MSB = c$, where a, b e c are positive constants and y is the quantity produced.

a) Represent graphically the social optimum and the market equilibrium.

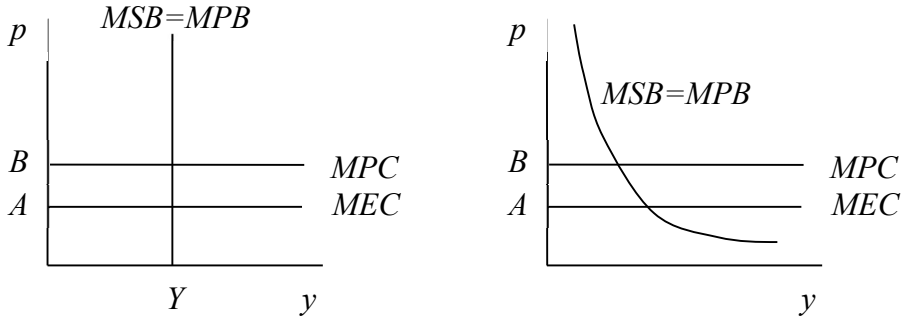
b) Determine the value of the Pigou tax.

Suppose there are two firms in this market, with marginal cost curves given by $MPC_1 = b_1 y$ and $MPC_2 = b_2 y$

c) What is the relation between b, b_1 e b_2?

d) What is the level of production for each firm when the Pigou tax is applied?

10) (Exam EE 11.07.08) Consider the following two possibilities for the market for electrical energy, where p is price and y is quantity of electrical energy:



In the second case,

$$MSB = MPB = \frac{a}{a+b}\frac{m}{y}$$

For each case, a Pigou tax is applied on the producer. For each of these two cases, calculate

a) the variation (between the initial situation and the situation after the Pigou tax is implemented) in
   i) electricity price,
   ii) electricity consumption,
   iii) externality,
   iv) consumer and producer surpluses,
   v) money spent by consumers on electricity,
b) the amount of government revenue raised by the tax.

25) (EA Exam 09.02.2008) Consider a firm that has the following marginal costs curve,

$$MC(y) = a(y-b)^2 + c$$

where *a*, *b* and *c* are positive constants.

a) Determine the firm's supply curve.
b) Consider a market with n firms and with the above marginal costs curve.
c) Determine the sector's (or industry) supply curve.
   Consider that in this market the demand curve is given by $p = Ay + b$, where *A* is negative and *b* positive.
d) Determine, in this curve, demand elasticity in relation to price.
e) Graphically represent the equilibrium in this market, consumer and producer surplus.
f) Determine the values of the variables graphically represented in the previous question. Interpret the results.

26) (EE Exam 24.01.2009) Consider a market where there are positive and negative externalities. The marginal social benefits and costs curves are respectively $MSC = 20 + Q$ and $MSB = 30 - 2Q$ (for $Q < 10$, being zero for $Q > 10$), where Q is the quantity produced. Positive and negative externalities are respectively given by $MEB = 10$ and $MEC = 20$. Thus, the marginal private benefits curve (that is, the good's demand curve to which this market is associated) is given by $MSB = 20 - 2Q$ (for $Q < 10$, being zero for $Q > 10$) and the marginal private costs curve (that is, the good's supply curve) is given by $MPC = Q$.

a) Determine whether the demand curve is elastic, inelastic or has unitary elasticity.
b) For the demand curve, determine the total cost, average cost and average variable cost curves. What is the value of fixed cost involved in this production process?
c) Determine market equilibrium. For this, determine and show graphically producer's surplus, consumer's surplus, external cost, external benefit and social balance.

d) Determine the social optimum, which is the point that maximizes the algebraic sum of producer's surplus, consumer's surplus, external cost and external benefit. For this, show graphically the sum of producer's and consumer's surplus, the external cost, the external benefit and social balance.

e) Determine and represent graphically the social gain of moving from the market equilibrium to the social optimum. Calculate the value of the Pigou tax that leads the market equilibrium to the social optimum.

27) Consider a market with a positive externality, resulting from the activity of producers, with $MPB = cy + d$, $MEB = ay$ and $MPC = MSC = by^2$, where *a*, *b* and -*c* are positive constants and *y* is the quantity produced.

a) Graphically represent the market equilibrium and the social optimum. For each of these represent the consumer and producer surpluses, the external benefit and the net social benefit. Also represent the social gain obtained with the transition from the market equilibrium to the social optimum.

b) Determine the value of the Pigou tax/subsidy.

Suppose there are two consumers in this market, with external benefit curves given by:

$MEB_1 = a_1 y_1$ and $MEB_2 = a_2 y_2$.

c) What is the relation between a, a_1 and a_2?

d) What is the level of consumption of each consumer, when the Pigou tax/subsidy is applied to the producers?

28) Consider a market with a positive externality, resulting from producers' activity, with $MPB = d - cy$, $MEB = ay$ and $MPC = MSC = by^2$, where *a*, *b* e *c* are positive constants, $a > c$, and *y* is the quantity produced.

a) Graphically represent the market equilibrium and the social optimum. For each represent consumer and producer surplus, external benefit and the social balance. Also represent the social gain obtained with the transition from market equilibrium to the social optimum.

b) Determine the value of the Pigou tax/subsidy.

Suppose there are two consumers in this market, with external benefit curves given by: $MEB_1 = a_1 y$ and $MEB_2 = a_2 y$.

c) What is the relation between a, a_1 and a_2?

d) Which is the level of consumption of each consumer, when the Pigou tax/subsidy is applied to the producers?

29) Consider the market for hybrid cars. The consumer demand for hybrid cars can be represented by $P/(€/car) = 400 - 2Q$, where Q is the number of cars (a non-dimensional quantity) The supply of hybrid cars is represented by the equation $P/(€/car) = 2Q$. The marginal environmental benefit of hybrid vehicles is 60 €/car.

a) Draw the demand curve, the marginal private cost curve and the marginal social benefit curve.

b) Determine the market equilibrium. Identify graphically and determine the consumer surplus, the producer surplus and the externality. Interpret the values obtained.

c) What are the socially efficient price and quantity of hybrid vehicles?

d) What is the amount of the optimal subsidy (Pigou subsidy) per hybrid vehicle? What price would consumers pay (after they get the subsidy) per vehicle? What price would suppliers of hybrids get? Justify your answers.

e) What is the change in consumer surplus, producer surplus and the externality between the market equilibrium and the social optimum? Identify on the graph. Based on these results discuss the benefits of introducing the subsidy.

30) (Env. Econ. Exam of 02.02.2011) Suppose a competitive economy exhibits the following supply (marginal costs) and demand (marginal benefit) curves respectively, $Q_S = 8P$ and $Q_D = 110 - 2P$. However, a production externality exists in the economy. The marginal social cost curve is given by $Q = 8P - 10$.

a) Compute the market and social equilibrium prices and quantities and represent them in a graphic.

b) Compute net social benefit in the social optimum vs. the competitive equilibrium. Identify the terms in net social benefit as areas in your graphic of the previous question.

How can you place the market economy on the social optimum? Discuss the benefits of introducing a tax or subsidy or whatever mechanism you choose to achieve the social optimum.

31) (Test EA 23.11.2011) Consider the market for alcoholic beverages. The national statistical office recently released a study about the supply and demand of alcoholic beverages divided by age groups. The total supply is described by the curve $Q_S = 2P$, where P is the price of alcoholic drinks. The demand for consumers between 18 and 40 years is described by $Q_D = 100 - 2P$. This age group of consumers of alcoholic drinks is associated with external costs to society due to an increase in road accidents. The marginal cost is estimated to be $MEC = 10$. When solving the questions below, summarize your results in a table of the form (t is the value of the tax, and Pd and Ps are respectively the prices facing the consumers and the produces):

	t = 0	t = 6	t = 12	t = ?
$Q^{Equi.}$				
$P_d^{Equi.}$				
$P_s^{Equi.}$				
CS				
PS				
Tax Rev.				
EC				
NSB				

a) Calculate the price and quantity of alcoholic drinks in the market equilibrium. What are the consumer surplus (CS), the producer surplus (PS) and the external cost (EC) in the market equilibrium? What is the net social benefit (NSB)? Draw the curves in a P-Q graphic and identify the areas corresponding to CS, PS, EC, and NSB.

b) The government, acknowledging a problem in road safety, decides to act and proposes a tax of $t = 12$ on the sales of alcoholic drinks. What would be the quantity and prices on the new market equilibrium, the PS, CS, government revenue and the EC associated with the government proposal? What is the NSB? Draw the demand and supply curves and identify the areas of the terms used to calculate NSB.

c) The producers of alcoholic drinks feel that they are being exaggeratedly penalized for the external cost to society of excessive drinking and instead propose that the government adopts a tax of $t = 6$. What would be the quantity and prices on the market equilibrium, the PS, CS, government revenue and EC? What is the NSB? Draw the curves and identify the areas of the terms used to calculate NSB.

d) The government is not satisfied with the proposal of the producers of alcoholic drinks and decides that it is convenient to create a commission to study this problem and come up with the most favourable tax level for the society. You are the head of this commission. What is your recommendation? Back your conclusions by presenting and analysing the summary table of the values of PS, CS, government revenue, EC and NSB for your proposal and for the proposals in a) b) and c). Which proposal maximizes net social gain? Draw the curves and identify the areas of the terms used to calculate NSB for your proposal. Identify the change in NSB from $t = 0$ to your proposal.

e) The government also needs to know the total demand for alcoholic drinks, and it obtains the information that for people older than 40 years the demand is described by $Q_D = 40 - P/2$. What is the expression for the total demand for alcoholic drinks? Draw in a graph the individual and total demand curves for alcoholic drinks.

32) (Test EA, 19.01.2012) A beekeeper has bees for their honey production. A side effect of his activity is the pollination of surrounding crops by the bees. The marginal private benefit, the supply and the marginal external benefit are, respectively, $P = 10 - Q$, $P - 2 = Q$ and $MEB = 4$€/kg, where Q is the amount of honey produced which is proportional to the number of bees. The honey is measured in kg and the prices are in €/kg.

a) What is the market equilibrium? At this point, calculate and represent graphically the consumer surplus, producer surplus and external benefit)?

b) Determine the social optimum. At the market equilibrium, is there under- or over-production of honey? Why?

c) Describe succinctly a mechanism (tax or subsidy) to bring the market equilibrium to the social optimum and determine the new market equilibrium. Represent the areas of the net private benefit, the external benefit and the net social benefit.

d) In the equilibrium attained in c), what is the price at which the beekeeper sells his honey? What is the price that the consumers pay for the honey?

e) Calculate the consumer and producer surpluses and external cost in the equilibrium attained in c). Calculate the change in net social benefit. Discuss the benefits of introducing the tax/subsidy you suggested.

f) Show in separate graphs the areas representing the change in net private benefit, net external cost and net social benefit when moving from the market equilibrium to the situation in c).

g) Assume that the above demand curve refers only to the production for national consumption of honey. The beekeeper is studying the possibility of exporting part of his production. He obtained the information that the foreign demand function for honey can be described by $P = 6 - Q/2$. If he decides to also sell abroad, what is the total demand for honey that he now faces? What is the new market equilibrium? What are the quantities of honey sold domestically and abroad?

33) (EA, Special Season, 12.07.2012) A farmer in Alentejo has a field of wheat and other cereals that he uses to sell regionally. The inverse demand (marginal private benefit) he faces is given by, $P = 10 - Q$, and he supplies the quantity Q of cereals according to the function $P - 2 = Q$, where P represents the price of a unit of cereals. Recently he has heard about ecosystem services and realized that his field can be considered to be a sink of CO_2 due to the accumulation of organic soil matter. Making some calculations using studies on the costs of carbon emissions and studies on absorption of carbon in cereal fields, he estimated the value of the absorption of carbon in his field to be around *4 €/kg* of cereals.

a) If a consumer is willing to pay 1 *€/kg* of cereals what is the quantity of cereals sold by the farmer? Why?

b) Determine the market equilibrium and the social optimum. Compare them and comment on their values (prices and quantities).

c) At the market equilibrium, calculate and represent graphically the consumer surplus, producer surplus and external benefit.

d) Assume the government is in favour of implementing a mechanism (applied to the demand side) to internalize the absorption of carbon in cereal fields. Describe succinctly a mechanism (tax or subsidy) to bring the market equilibrium to the social optimum. In the new market equilibrium what is the price at which the farmer sells his product? What is the price that the consumers pay?

e) Calculate the consumer and producer surpluses and external cost in the equilibrium attained in d). Calculate the change in net social benefit. Discuss the benefits of introducing the tax/subsidy you suggested.

f) Show in separate graphs the areas representing the change in net private benefit, net external cost and net social benefit when moving from the market equilibrium to the situation in d).

g) What happens to the net social benefit (compared to your answer in e)) if the tax/subsidy is higher or lower than the tax/subsidy you proposed in d)? Why?

Assume that the above private demand curve refers only to the demand for cereals in Alentejo.

h) The farmer is considering the possibility of selling at the national level, where it is known that the Portuguese demand function for cereals can be described by $P = 8 - Q/2$. If the farmer decides to sell at the national level, what is the total demand for cereals that he now faces? What is the new market equilibrium (also represent graphically)? What are the quantities of cereals sold regionally and nationally?

34) Consider the inverse demand function $p = a - bq$, where p is price, q is quantity and a and b are positive constants.

a) Calculate the elasticity. State the ranges of quantity where it is elastic, inelastic or unit elastic.

Consider the inverse supply function $p = c + dq$, with $c>0$, $d>0$.

b) Calculate the market equilibrium (price and quantity). Interpret.

Consider now that $c = 0$, and consider a positive externality given by $MEB = g$.

c) Interpret the condition $c = 0$. Interpret the price and quantity at the market equilibrium under this condition.

d) Represent graphically and calculate the consumer and producer surpluses and the external benefit at the market equilibrium.

e) Calculate the social optimum. Interpret its relation to the market equilibrium, and the effect of the parameters on this relation.

f) Calculate the Pigou subsidy which should be paid to the producer.

g) Represent graphically the consumer surplus, the producer surplus, the externality and the total amount of subsidy paid by the government in the new market equilibrium, attained after applying the Pigou subsidy.

h) Represent graphically the change in social welfare when going from the original market equilibrium to the market equilibrium with the Pigou subsidy.

35) As explained in the classes for this course, producer theory can be understood in complete symmetry with consumer theory. So, consider the Cobb-Douglas production function,

$$f(x_1, x_2) = kx_1^{\alpha} x_2^{\beta},$$

which gives the quantity produced, as a function of the quantities used of each production factor, x_1 and x_2, with positive α and β and $\alpha + \beta = 1$.

a) Sketch on the x_1 - x_2 plane the family of isoquants associated to this production function, i.e., the lines of constant production (constant f).
Consider the producer's cost minimisation problem:

$$\min w_1 x_1 + w_2 x_2$$
$$s.t. kx_1^{\alpha} x_2^{\beta} = y$$

where $w_1 x_1 + w_2 x_2$ expresses the cost of production of a quantity y.

b) Calculate the amounts of x_1 and x_2 that minimise the cost, for a certain production level y. Interpret the result.

c) Calculate the resulting cost, the optimal $w_1 x_1 + w_2 x_2$, as a function of y, i.e. $C(y)$. Calculate the marginal cost. Draw graphs of cost and marginal cost. Interpret.

d) Calculate the fraction of the cost spent on each production factor.

For the rest of this problem, if you were not able to complete question c), assume that marginal cost is constant, equal to K.

Suppose that the producer is producing for a market with demand function $p = a - by$, with a and b being positive constants.

e) Calculate the price elasticity of demand for this demand function. Determine the quantity ranges where it is elastic, inelastic and unit elastic.

f) Calculate and represent graphically the market equilibrium. At the market equilibrium, represent the consumer surplus and the producer surplus. Interpret.

Suppose that this production process consumes water, generating an externality given by MEC = a.

g) Represent graphically the net private benefit (consumer surplus + producer surplus) and the externality at the social optimum. Interpret, by comparing this situation to the market equilibrium.

h) Determine the Pigou tax which would be required to internalise this cost.

i) Represent graphically, with justification, the change in net social benefit due to the imposition of a Pigou tax.

36) (Env. Econ. Exam, 18.01.2014) Consider a market with a negative externality, with $MPC = by$, $MEC = ay$, $MPB = MSB = c - dy$, where a, b, c and d are positive constants and y is the produced quantity.

a) Represent graphically and calculate the market equilibrium in perfect competition, indicating graphically the producer and the consumer surpluses and the externality.

b) Represent graphically and calculate the social optimum. Justify whether the net private benefit is higher or lower at the social optimum than at the market equilibrium.

   i) Suppose the government had the power over this market to decide both prices and quantities. Suppose the government decides that the quantity produced and bought has to be the socially optimum, and that the price is equal to

   ii) $\frac{bc}{a+b+d}$.

   iii) Note that this is a mechanism to attain the social optimum where the government neither charges a tax nor provides a subsidy.

c) Justifying with calculations or graphical representations, state whether:

   i) the producer surplus in this situation is higher or lower than if the government had applied a Pigou tax on producers and let the market operate freely;

   ii) the consumer surplus in this situation is higher or lower than if the government had applied a Pigou tax on producers and let the market operate freely;

iii) the external cost in this situation is higher or lower than if the government had applied a Pigou tax on producers and let the market operate freely.

37) (Env. Econ. Exam, 27.01.2014) Consider a Cobb-Douglas consumer with demand curves given by

$y_1 = \frac{a}{a+b}\frac{m}{p_1}$ and $y_2 = \frac{a}{a+b}\frac{m}{p_2}$

Suppose that the supply curves for these goods are given by respectively $MPC_1 = c_1$ and $MPC_2 = c_2$. Suppose additionally that the production of good 1 is dirty, leading to a negative externality given by $MEC_1 = d_1$, whilst the production of good 2 is clean, with no negative externality.
For good 1:

a) Calculate and represent graphically the market equilibrium.

b) Calculate and represent graphically the external cost at the market equilibrium.

c) Calculate and represent graphically the social optimum.

d) Calculate and represent graphically the external cost at the social optimum. Is it higher or lower than at the market equilibrium? Interpret.

e) Calculate the Pigou tax and the government revenue obtained with it. Comment on the relation between the government revenue obtained and the external cost at the social optimum.

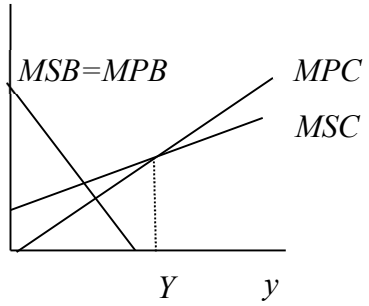
We now explore what the government might do with the revenue obtained, hence obtaining a more complete view of the effect of the Pigou tax. Assume that the government uses the revenue to subsidise the clean good, by giving a unit subsidy equal to *s* to the producers of good 2.

f) Calculate the value of *s* that leads the government to completely spend on this subsidy the revenue obtained with the Pigou tax.

g) Represent graphically the total subsidy given by the government and the corresponding change in consumer surplus regarding good 2. Comment on the relation between the two and on its contribution to an overall assessment of the effect of the Pigou tax.

38) (EA Exam 17.01.2015). Explain and give examples, graphical and relating to real life situations, of when it is optimal for society to have zero pollution and when it is not.

39) (EA Exam 17.01.2015). Consider a market with a negative externality, characterised by the following curves:



a) Represent graphically the market equilibrium in perfect competition, indicating graphically the producer and the consumer surpluses and the externality.

b) Represent graphically the social optimum. Justify whether the net private benefit is higher at the social optimum or at the market equilibrium.

c) Interpret the difference between situations for which $y < Y$ and $y > Y$.

40) (EA Exam 18.01.2016) Consider a market with demand curve given by $MPB = b - cy$, supply curve given by $MPC = a$, and a negative externality given by $MEC = d$ where a, b, c and d are positive constants, with $b > a + d$.

a) Calculate the price-demand elasticity of the demand curve, and state whether it is elastic, inelastic or unit-elastic.

b) Consider this market is under perfect competition. Calculate the price and quantity at equilibrium. Represent graphically the producer surplus, the consumer surplus and the external cost at the market equilibrium.

c) Consider instead that the there is only one producer, a monopolist, who is free to set the price p at which he sells.

i) At price p, how much is his revenue and his profit?

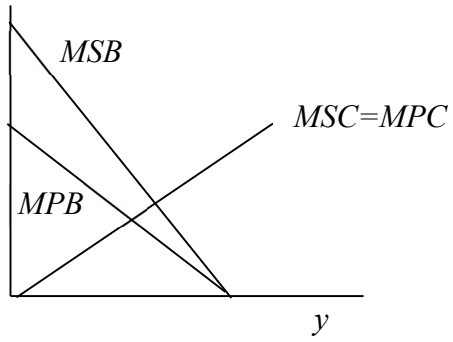
ii) Calculate the prices that maximize his revenue and his profit. Interpret the results, considering also the results of a) and b).

d) Note that $b > a$. Interpret this condition.

e) Show that the optimal price for the monopolist is higher than the price established under perfect competition.

f) Represent the consumer surplus, the producer surplus and the external cost at the optimal point for the monopolist and compare with the same quantities under perfect competition (represented in question b)). Interpret.

41) (EA Exam 26.01.2015) Consider a market with a positive externality, characterised by the following curves:



a) Represent graphically the market equilibrium in perfect competition, indicating graphically the producer and the consumer surpluses and the externality.

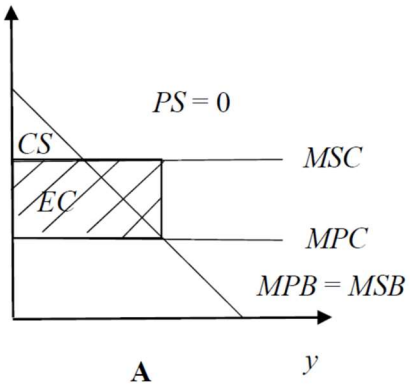
b) Represent graphically the social optimum. Explain the meaning of this point.

c) Justify whether the net private benefit is higher at the social optimum or at the market equilibrium. Interpret.

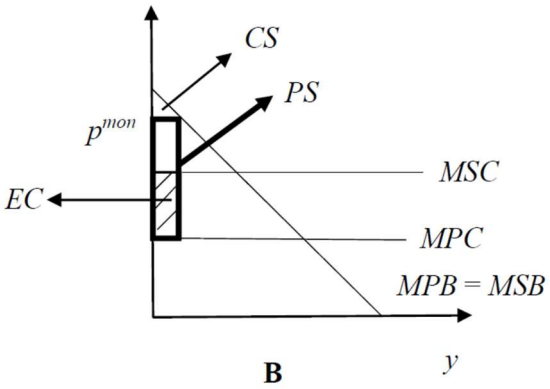
d) Represent graphically, with justification, the difference in net social benefit between the market equilibrium and the social optimum.

e) Give examples of production processes and environmental issues that might be in accordance with this graph.

42) (EA Exam 02.02.2016) Consider a market with demand curve given by $MPB = b - cy$, supply curve given by $MPC = a$, and a negative externality given by $MEC = d$ where a, b, c and d are positive constants, with $b > a + d$. Consider that there are two possible market structures: a monopolist (i.e., the supply curve presented is that of a single producer); perfect competition (i.e., the supply curve presented is the aggregate of many producers). Suppose the monopolist chooses to sell at price p^{mon}. The following graphs A and B present the consumer surplus, the producer surplus and the externality in both situations:



A



B

From the point of view of the ethical theories of utilitarianism, libertarianism, Kant and Rawls, discuss whether the monopolist should be allowed to maintain his activity, or whether he should be forced to break up into small companies (like, e. g. happened in the famous case of Standard Oil, in the USA in the beginning of the 20th century), hence leading to a market structure as presented in B (max. 300 words).

43) (EARN Exam 14.01.2017) Consider the Cobb-Douglas utility function for an individual consumer given by $u(x, y) = kx^a y^b$, with k, a and b positive constants, with x being gasoline and y being a composite good, comprising all other goods.

a) Given the budget constraint $p_1 x + p_2 y = m$, calculate the optimal amounts of x and y.

b) Consider now that there are n individuals in the population, all with this utility function. Calculate the aggregate demand functions for these two goods, with total quantities given by X and Y.

Suppose the production of gasoline has marginal production cost given by $= MPC(y) = g$, where g is a positive constant.

c) Calculate the quantity of gasoline consumed at the market equilibrium. Interpret the result.

The use of gasoline generates pollution. Suppose this has marginal external cost given by $MEC = c$, with c a positive constant.

d) What is the optimal level of pollution in the market for gasoline? Interpret the result.

Because of this pollution, the government has imposed a Pigou tax.

e) What is the consumption of gasoline in the presence of a Pigou tax?

f) What is the consumption of the composite good in the presence of the Pigou tax? Interpret.

44) State and justify succinctly which of these statements are correct and incorrect:

a) A utilitarian believes the Pigou tax should be applied because it increases the sum of welfare for everyone in society.

b) A libertarian is against a Pigou tax because its payment is not voluntary, it is coerced.

c) A utilitarian believes a Pigou tax should not be applied if the government uses wastefully the tax revenue it receives.

45) (EA Exam 01.02.2017) Explain what the Pigou tax is, what is the problem it solves and how does it solve the problem; present the appropriate graphs to illustrate your answer.

46) (ENRE Exam 12.01.2018) Traffic in large cities is often a major problem, both due to pollution from cars and the time wasted stuck in traffic jams. Let us define x as "time spent driving in Lisbon" and C is all other consumption (measured in money). Assume that utility is given by

$$u(x, C) = \left(ax - \frac{b}{2}x^2\right) + dC$$

where a, b and d are positive constants. Drivers have income m and the cost of driving (due to gasoline consumption, etc.) is px.

a) Calculate the marginal utilities of driving and other consumption.

b) Is driving a good or a bad? Interpret.

c) Sketch the indifference curves for this utility function.

d) Write the budget constraint and represent it on the graph in c).

e) Check that the demand function for x is $x = (a - dp)/b$. Interpret.

f) Estimate, with justification, a plausible (in terms of order of magnitude) value for p; choose the particular number to facilitate the calculation in g).

g) Considering a = 100/(minute/week), b = 2/(minute/week)^2 and your estimate of p, how much time will someone drive in Lisbon per week? Is this number plausible?

Assume the external costs of driving in cities are given by (with k a positive constant):

$$EC(x) = kx^2$$

h) Represent on the same graph, the lines for marginal private benefit, the marginal social benefit, the marginal external cost and the marginal social cost.

i) Assume that p=0. On the same graph, represent the time spent driving. For this point, represent the consumer surplus and the external cost.

j) Again on the same graph, represent the optimal time spent driving, from society's point of view. Again, represent the consumer surplus and the external cost. Interpret, comparing with i).

47) (ENRE Exam, 31.01.2018) Consider a market where a certain good is produced and consumed in a quantity y, with $MPC = ay$, $MEC = by$, $MPB = MSB = c - dy$, where a, b, c and d, are positive constants.

a) Represent graphically and calculate the consumer surplus, the producer surplus and the externality at the market equilibrium.

b) Calculate the social optimum and the Pigou tax which takes the market equilibrium to this social optimum.

c) Represent graphically the consumer surplus, the producer surplus, the externality and the total amount of tax paid, after a Pigou tax is applied on the producers.

d) Represent graphically the consumer surplus, the producer surplus, the externality and the total amount of tax paid, after a Pigou tax is applied on the consumers.

e) Compare graphically the prices before and after the Pigou tax is applied and, considering c) and d), discuss the statement: "producers should bear the cost of environmental taxes, because they are causing the pollution".

48) (EE Exam 23.01.2019) Consider the following quasilinear utility function, where y is consumption of electricity and x comprises all other goods:

$$u = ay + bln\, x/x_0$$

a) Sketch the family of indifference curves for this utility function.

b) Calculate the marginal utilities. Are x and y goods or bads? How is this related to the marginal utilities? Relate this to what x and y are.

c) Calculate the marginal rate of substitution (MRS). Interpret the result, considering the economic meaning of the MRS and your answer to b).

Consider the consumer faces the following budget constraint:

$$p_1 x + p_2 y = m$$

d) Plot this budget constraint on the graph in a).

e) Calculate the optimal choices of x and y for this consumer (taking into account that there is a corner solution). Interpret (namely considering the meaning of x and y).

Suppose now there are n consumers, all with the same utility function, but different incomes, m_i, i=1,…,n.

f) Calculate the market demand curve for electricity of this set of consumers, ignoring the corner solution.

g) Calculate the inverse demand curve, for the situation in f).

Suppose now the supply of electricity is given by MPC = c.

h) Calculate and represent graphically the equilibrium quantity and price in the market for electricity. State the assumption that is required to guarantee that this equilibrium exists.

Suppose a tax per unit produced is imposed on producers, with value equal to t.

i) Calculate the new market equilibrium. Interpret the effect of t on this market equilibrium (this should include stating the conditions on t for the market equilibrium to still exist).

Consider now that the generation of electricity produces an externality, due to the emission of carbon dioxide, with a constant marginal external cost equal to π.

j) Calculate the level of production which is socially optimal and represent it in the same graph as in h). Interpret.

Suppose now that the tax t is chosen to be a Pigou tax, i.e., a tax which takes the market equilibrium to the social optimum.

k) Calculate t.

49) (EE Exam 01.02.2019) Consider the following quasilinear utility function, where y is consumption of electricity and x comprises all other goods:

$$u = ay + b\ln x/x_0$$

For a consumer with budget constraint

$$p_1 x + p_2 y = m$$

the demand function is (for $m \geq bp_2/a$)

$$\begin{cases} x = \dfrac{b}{a}\dfrac{p_2}{p_1} \\ y = \dfrac{m}{p_2} - \dfrac{b}{a} \end{cases}$$

Suppose now there are n consumers, all with the same utility function, but different incomes, m_i, i=1,…,n.

a) Calculate the market demand curve for electricity of this set of consumers.

b) Calculate the inverse demand function (equal to marginal private benefit).

Suppose now the marginal private cost of supplying electricity is constant and equal to c.

c) What is the inverse supply function?

d) Calculate the price and quantity at the market equilibrium. Interpret.

Consider the net private benefit, NPB, equal to the private benefit minus the private cost (i.e., the consumer surplus plus the producer surplus).

e) Calculate the quantity y that maximises NPB. Interpret.

Consider now that the generation of electricity produces an externality, due to the emission of carbon dioxide, with a constant marginal external cost equal to π.

f) Calculate the level of production which is socially optimal. Interpret.

g) What is the relation between NPB at the market equilibrium and at the social optimum? Interpret.

Discount Rate

1) (EE Exam 12.01.2005). The following table presents the interest and inflation rates for the USA.

	1965	1970	1975	1978
Inflation rate	2,9	4,3	4,2	11,3
Nominal interest rate (%)	4,0	6,4	5,8	7,2
Real interest rate	1,1	2,1	1,6	

a) Complete this table

b) By the end of the 70's, Americans complained about high interest rates which had never been as high as recently. Explain why these complaints were injustified.

2) What is the present value of 100€ one year from now if the interest rate is 10%? What is the present value if the interest rate is 5%?

3) If inflation increases:

   a) the discount rate increases;

   b) the discount rate remains constant;

   c) the discount rate decreases;

   d) it is not possible to establish a link between changes in inflation and discount rates.

4) If I deposit in a bank 1000 € today and receive 1050 € in a year from now:

   a) I lost money because there exists another bank which would give me 1070 € a year from now;

   b) I lost money because the inflation rate is 3%;

   c) I won money because the inflation rate is 3%;

   d) none of the above is correct.

5) In general, when the average growth rate of consumption increases:

   a) the discount rate increases;

   b) the discount rate remains constant;

   c) the discount rate decreases;

   d) it is not possible to establish a connection between changes in the average growth rate of consumption and changes in the discount rate

6) Assuming that the effect on utility of a variation of income does not depend on income, if there is an increase in the consumption growth rate:

   a) the discount rate increases;

   b) the discount rate remains constant;

   c) the discount rate decreases;

   d) it is not possible to establish a connection between changes in the average growth rate of consumption and changes in the discount rate

7) If we expect to be in the future poorer than we are today:

   a) the discount rate of consumption will surely be positive;

   b) the discount rate of consumption may be positive or negative;

   c) the discount rate of consumption will surely be negative;

   d) the discount rate of consumption only depends on our impatience towards the future, being independent of our wealth in the future;

8) (EE Exam 13.09.2005). Calculate the current value of a benefit of 10 000 € occurring 5 years from now, considering a 5% discount rate.

9) Supose the expected costs of a project of polution control are 8 million euros per year, and the benefits are 50 millions per year in the next 50 years and 150 millions euros per year thereafter. Using a 4% discount rate what are the net benefits of the

project? What are the net benefits for a 2% discount rate? Comment on the difference in the results.

Note: Consider a succession $u_1, ..., u_n$, with $u_i / u_{i-1} = r$. Then, $\sum_{i=1}^{n} u_i = u_1 \frac{1-r^n}{1-r}$.

1) (EE Exam 13.01.2006). Calculate the present value of a payment of 350 € in 7 years from now, considering three consumption discount rates: 6%, 8%, and 10%. For each case, what is the maximum value you would be willing to pay now to receive this payment in 7 years?

2) (EE Exam 25.01.2006). Consider the projects A and B whose net benefits in time are given by the following table:

Project	1	2	3	4
A	60	20	20	10
B	30	30	30	30

For a discount rate in discrete time of 5%, determine which Project should be selected.

3) (EE Exam 12.01.2007). Consider an 18-year-old young man who is deciding whether to enlist in a graduate degree. Assume that if he does not enlist he will work from 18 to 65 years old with an annual income of 10.000 €. Alternatively, he can graduate supporting the annual costs of 1.500 € during the 5 years of study and works from 23 to 65 years old with an annual income of 15.000 €. Using a discount rate of 3% which alternative should the young man choose? Discuss the result.

4) Suppose that Concha's preferences regarding intertemporal consumption are described according by $U = \ln C_1 + \frac{\ln C_2}{1,075}$. The income in periods 1 and 2 is respectively equal to m_1 e m_2. Concha may wish to lend or borrow money with an interest rate of 7.5%.

   e) Write the expression for the intertemporal budget constraint.

   f) Represent graphically the solution of Concha's optimization problem.

   g) Calculate the consumption chosen by Concha in periods 1 and 2.

   h) Will Concha be a lender or a borrower in period 1? Calculate the amount of money lent or borrowed.

National Accounts

[Recommended Reading: UN (2003). *National Accounts: A Practical Introduction*. Studies in Methods Series F, No.85 - Handbook of National Accounting.
http://unstats.un.org/unsd/publication/SeriesF/seriesF_85.pdf
Chapter 1; Chapter 2 – A, B, D (except paragraphs 2.57-2.59 and 2.64 – 2.66).]

1) (ENRE 18.01.2019). Explain how the price of labour, i.e., the wage, is formed in the view of the Classical economists.
2) (ENRE 18.01.2019). In the last decades GDP has been used by most countries to assess the size and the prosperity of their economies. Present three main advantages of GDP that justify its universal acceptance.

3) (ENRE Exam, 05.02.2019). Since its creation, GDP has been subject to much criticism. Name three shortcomings. Explain why they are indeed shortcomings and how they could be overcome.
4) (EE Exam, 23.01.2019). Under what conditions can GDP be increasing but GDP per capita be decreasing?
5) (EE Exam, 23.01.2019). Under what conditions can GDP per capita be increasing but total payments to labour (i.e., total salaries paid) be decreasing?
6) Why is it desirable for a country to have a large GDP? Give an example of something that would raise GDP and yet be undesirable.
7) A farmer sells wheat to a baker for 10 €. The baker uses the wheat to make bread, which is sold for 15 €. What is the total contribution of these transactions to GDP?
8) List the four components of GDP following the expenditure (or demand) approach and give examples of each component.
9) Consider the GDP and GNI (gross national income or product). Which measure should a government prefer if it cares about the total income of Portuguese population? Which should it prefer if it cares about the total economic activity occurring in Portugal? Why?
10) State whether the following propositions are true or false and why. In the case of false propositions what would be the correct versions:
    a) The value added of a firm owned by Portuguese residents and functioning on Portuguese territory is part of the Portuguese GDP and GNI.
    b) The wage of a resident who during 4 months worked for a firm based in Spain is a part of Spanish GNI and Portuguese GDP.
    c) The operating surplus (profits) – capital remuneration of a firm located in Portugal but owned by Germans – sent to Germany, is part of the Portuguese GDP and the German GNI.
    d) The income earned by Portuguese emigrants working abroad as residents is part of the Portuguese GDP and GNI.
11) Consider the economy of Fakeland. In this country the System of National Accounts is still being organized and it is currently incomplete. For the year 2009, the following data is available (where ROW is rest of the World):

    Total production: 30 500
    Intermediate consumption: 11 200
    Gross Investment (or gross capital formation): 5 305
    Taxes net of subsidies on products: 1 800
    Consumption of fixed capital: 4 000
    Final consumption expenditure: 12 500
    Total imports: 6 300
    Income flows from ROW minus income flows to ROW: - 5 000
    Current transfers from ROW minus current transfers to ROW: 100

    a) What was the GDP of Fakeland in 2009?
    b) What was the value of the total exports of Fakeland in 2009?
    c) What was the gross saving and net saving of Fakeland in 2009?
    d) When discussing the financial sustainability of Fakeland the government focuses its attention on Gross Saving. However, some of the opposition is basing its arguments for a change in economic policy by looking at Net Saving. What is the more prudent approach and why?
    e) What would you suggest to increase Fakeland's Net Saving?
12) Suppose that the residents of Vegopolis spend all of their income on cauliflower, broccoli, and carrots. In 2001 they buy 100 heads of cauliflower for 200€, 50 bunches of broccoli for 75€, and 500 carrots for 50€. In 2002 they buy 75 heads of cauliflower for 225€, 80 bunches of broccoli for 120€, and 500 carrots for 100€. If

the base year is 2001, what is the CPI in both years? What is the inflation rate in 2002?

13) Consider the following table:

Year	Nominal GDP	GDP Deflator (Base year 2000)
2005	7 662	110
2006	8 111	112

i) What was the growth rate of nominal GDP between 2005 and 2006?
ii) What was the growth rate of the GDP deflator between 2005 and 2006?
iii) What was the real GDP in 2005 and 2006 measured in 2000 prices?
iv) What was the growth rate of real GDP between 2005 and 2006?
v) Was the growth rate of nominal GDP higher or lower than the growth rate of real GDP? Explain.
vi) If prices rise, one effect is that people's revenue from selling goods increases. The growth of real GDP ignores this particular gain, however. Why, then, do economists prefer real GDP as a measure of economic well-being?

14) (Exam of EA 02.02.2011) Assume Springfield is a closed economy that only produces two types of goods, Krusty burgers and doughnuts. Below are some data regarding the price and the quantity of Krusty burgers and doughnuts produced or consumed.

Year	Price per burger [€]	Quantity of burgers	Price of doughnuts [€]	Quantity of doughnuts
2001	1	100	2	50
2002	1	200	2	100
2003	2	200	4	100

Assume that the basket used to estimate inflation in Springfield is defined as: 2 Krusty burgers and 1 doughnut. Use 2001 as the base year. Calculate the inflation rate in 2002 and 2003.

15) Consider four firms: firm A, a mining enterprise, extracts iron ore; firm B, a steelmaker, uses iron to make steel sheets and ingots; firm C, a carmaker, makes automobiles using steel; firm D, a manufacturer of machinery and robots, also uses steel.

i) Calculate the production, intermediate consumption and values added in millions of euros based on the following assumptions.

Firm A extracts 50 000 tons of ore, at 200 euros per ton, its purchases during the period are limited to the purchase of one machine made by firm D, costing 10 million euros. Firm B produces 15 000 tons of steel sheet at 3 000 euros per ton, having bought and used all the ore produced by firm A. Firm C has manufactured 5 000 vehicles and sold them all to households for 15 000 euros each, having purchased 20 million euros' worth of steel sheet from firm B, but using only 18 million euros' worth in the manufacture of its cars. In addition, Firm C imported 5 000 engines from a foreign subsidiary, each being valued at 4 000 euros, and purchased domestically 2 robots made by firm D. Firm D sold one machine for 10 million euros

and two robots, each worth 5 million euros, having used 10 million euros' worth of steel sheet from firm B.

ii) Calculate the GDP of this economy. Calculate also the final demand of this economy, assuming that it has no exports. Verify that GDP is equal to final demand. (Remember that purchases of machinery are not intermediate consumption, but Gross Fixed Capital Formation).

iii) Let us now suppose that we omitted to mention that firm C, the car maker, hired manpower from firm E, the temporary employment agency, for the sum of 15 million euros. Has the GDP of the economy been modified by this fresh information? Confirm your reply by reconstituting the table for the different industries, with comments.

16) Consider the barber shop Hairy Harry, Inc. In one day, Hairy Harry Inc. collects €400 for haircuts. Over this day, his equipment depreciates in value by €50. Of the remaining €350, Harry sends €30 to the government in sales taxes, takes home €220 in wages, and retains €100 in his business to add new equipment in the future. From the €220 that Harry takes home, he pays €70 in income taxes. Based on this information, compute Harry's contribution to the following measures of income:

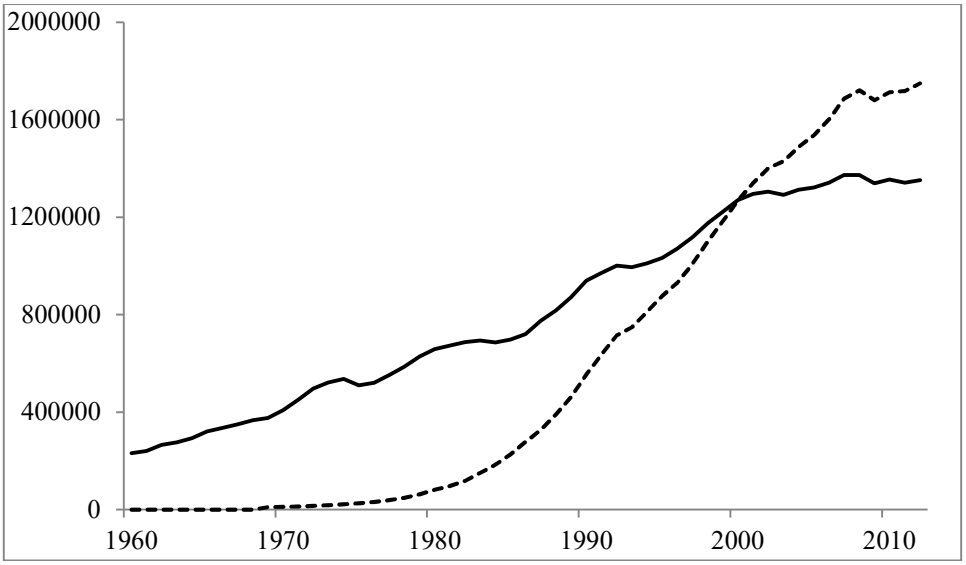
i) Gross Domestic Product
ii) Net National Product
iii) National Income
iv) Disposable Income

17) (EA Test 23.11.2011) Consider the following information on the functioning of an economy of a given country in a given year:

Private consumption: 600
Output: 1270
Public consumption (Government Expenditures): 160
Intermediate Consumption: 300
Gross Investment (Gross Fixed Capital Formation): 480
Exports: 310
Imports: 380
Income payments <u>from</u> the rest of the world (ROW): 80
Taxes net of subsidies on production: 200
Consumption of Fixed Capital: 140
Income payments <u>to</u> the ROW: 60
Transfers from the government to the households: 200
Net transfers <u>from</u> the ROW: 60

a) What is the value of Gross Domestic Product (GDP) of this economy calculated by the output and the demand approaches?
b) What is the value of Net National Income (NNI)?
c) At the end of the year, does the government have a deficit or a surplus? What is its value?
d) Give one example of a real-life money flow that could be considered an income payment <u>to</u> the rest of the world (ROW) and one example of a transfer <u>from</u> the ROW.

18) (EA Test 23.11.2011) The following figure shows the nominal and real GDP for Portugal in billions of euros. Identify both time series. How could you identify the base year used in calculating the real GDP? Justify.



19) (EA Test 19.01.2012) You are a Secretary of State for the Ministry of Labour and you are currently negotiating changes in the wages of the public sector. Below is some data regarding the price and the quantity for the most important consumption items. Use 2009 as the base year for your calculations.

Year	Price of electricity [€/kWh]	Quantity of Electricity [kWh]	Price of Water [€/m^3]	Quantity of Water [m^3]
2009	1	50	2	100
2010	0.7	100	3	200
2011	0.5	100	4	200

a) The government needs information on the inflation rate to negotiate potential increases in wages and you decide to compute the Consumer Price Index (CPI) using a basket composed of 200 kWh and 100 m^3. What is the CPI in each year? What is the inflation rate in 2010 and 2011? What is your recommendation for a wage change and why?

b) The public servants are enraged with you and your minister stating that you did not take into account the changes in quality-of-life over the years, and urge you to recalculate your estimates of inflation rate with a more adequate basket. Choose a better basket and recalculate the inflation rate in 2010 and 2011 taking into account the public servants' demands and worries. What is the inflation rate in 2010 and 2011? What is now your recommendation for a wage change and why?

20) (EA Special Season, 12.07.2012) Comment, briefly justifying (max 4 lines each), on the veracity, or not, of the following statements:

a) A Portuguese emigrant working in Germany saves part of his earnings to send to his family in Portugal. This money flow is accounted in the Portuguese GNI as a payment to factors of production.

b) The Portuguese football team Sporting C.P. was invited to play a friendly game in Angola and received a monetary compensation for it. This payment should be included in the Portuguese GDP but not in the Portuguese GNP.

c) Increasing the final consumption increases GDP and consequently the gross national savings.

d) Increasing the investment increases the net national savings, all else equal.

e) Real GDP is growing faster than nominal GDP because the price level is increasing, i.e., there is inflation.
f) The Portuguese Consumer Price Index (CPI) was 100 in 1983 and is around 210 in 2012. This means that 10 € in 1983 are equivalent of 12.1 € in 2012.
g) For the same wage, the amount of consumption goods bought is higher in Faro than in Coimbra, given that, respectively, their CPIs are 140 and 132.

21) (ENRE Exam, 18.01.2019). Explain the differences between using the CPI (Consumer Price Index) and the GDP deflator to remove inflation when comparing the GDP in two different years for the same country.

22) (ENRE Exam, 18.01.2019). Why is PPP (purchasing power parity) used in cross-country comparisons of GDP?

Economic Growth

1) (SDEE-M Exam, 12.01.2019). Which are the two conventional primary factors of production in the economy? Discuss briefly each of them, and how they can be measured.

2) (SDEE-M Exam, 28.01.2019). Describe what is represented by the aggregate production function.

3) (EE Exam, 01.02.2019) There are three ways in which GDP can measured. Let us consider two:
Income measure: GDP = $rK+ wL$,
Expenditure measure: GDP = $C + I$,
where w is average wage, L is labour, r is average interest rate, K is capital, C is consumption.
On the other hand, GDP is a function of capital and labour, GDP = f(K,L).
Suppose we take the aggregate production as being the Cobb-Douglas production function with constant returns to scale:

$$GDP = AK^a L^b$$

a) What is the relation between a and b?

b) What is the relation along time between I and K?

Cost minimisation, using the Cobb-Douglas production function, tells us that

$$\frac{rK}{GDP} = a, \qquad \frac{wL}{GDP} = b$$

c) One of the well-known Kaldor stylised facts of economic growth tells us that r is approximately constant along time. What does this imply regarding the K/GDP ratio along time?

d) The most salient feature of economic growth is possibly that w increases along time. What does this imply regarding the L/GDP ratio? Interpret.

4) (SDEE-M Exam, 28.01.2019). Total factor productivity is often called a "residual" component of economic growth. Explain why. Why is it also often called "a measure of ignorance "?

5) (SDEE Exam, 07.01.2016) The main factor for economic growth is that

a) part of economic output is invested in capital, and so the capital stock grows with time;

b) workers increase their working time;

c) each unit of capital and each unit of labour are increasingly productive along time.

6) (SDEE Exam, 09.01.2018). Economic growth happens because

a) entrepreneurs find ways of combining technological developments and management innovation in new business models;

b) the real cost of capital is decreasing along time;

c) a fixed amount of useful exergy is required per unit of economic value;

d) the amount of capital increases along time.

7) Consider a Solow-model economy at steady state. Describe what happens to this economy and represent in a graphic of $\left(k,\dot{k}\right)$ what, if at all, is the line of the following developments:

e) The rate of depreciation falls.
f) The rate of technological progress rises.
g) The rate of labour-force growth rises.
h) The savings rate falls.

1) Consider the Solow growth model without population growth and technological progress. The depreciation rate is 0.05. The production function is given by $Y = K^{1/3}L^{2/3}$.

a) Rewrite the production function in per-worker terms.

b) Write and interpret the expression that governs the motion of accumulation of capital per worker. What is the consumption level of this economy?

c) Identify the actual investment level and the break-even investment in the plane $\left(k,\dot{k}\right)$. If the savings rate *s* is 0.2, calculate the capital stock in the steady state, k^*?

d) Explain why the economy converges to a constant growth path when time goes to infinity whatever the initial capital stock. Show that the growth rate of output per worker declines as the economy approaches the steady state.

e) In what sense does the Solow model predict that poor countries grow faster?

f) Let us say a benevolent social planner wishes to maximize consumption per capita in the steady state. What is the stock of capital necessary to achieve maximum consumption? What is the savings rate that must be imposed by the social planner to support this capital level?

g) Compare this savings rate with the assumed rate in the question c) and give an intuition to this result. (Hint: What happens to consumption today and consumption in the future?)

h) In terms of economic welfare is it possible to save too much according to the Solow model? Explain.

8) (EARN Exam 17.01.2020) The newly-appointed government of the island-nation of Genosha is implementing the System of National Accounts in its economy, composed of five sectors: households, firms, government, banks, and the rest of the world (ROW). For the year 2019, the following statistics have been gathered, in nominal terms:

Private consumption: 470
Total consumption: 800
Taxes net of subsidies on private consumption: 200
Investment: 550
Trade Balance: -600
Net primary income flows to ROW: 85
Current net transfers from ROW: 65
Consumption of Fixed Capital: 250

a) Assuming that in 2019 the Genoshan government had a surplus of 100, and that firms are not subsidized, how much was collected in taxes on production that year?
b) Calculate the Gross Domestic Product of Genosha for the year 2019, using the expenditure approach. Determine the Gross Value Added for that year. What approach did you use to obtain the GVA?
c) From the available information, compute GNI, GNDI, and Net Savings. Define each of these quantities and what they represent. Illustrate your answer with examples.
d) In 2019, the total stock of capital K available to production in Genosha was 10500. According to the perpetual inventory method, what will be the projected total capital stock available in 2020?
e) The new government of Genosha is boasting that GDP will grow by 3% in 2020. Meanwhile, the Consumer Price Index in 2020 is expected to be 102. Knowing that the year 2019 was fixed as the base year for the CPI of the country, how much is the Genoshan economy expected to grow between 2019 and 2020 in real terms, and what will be the inflation rate?
f) The relationship between the level of output and the combination of capital K and human labour inputs L to production for the Genoshan economy can be represented by a Cobb-Douglas aggregate production function

$$Y = A \cdot K^{0.3} \cdot L^{0.7}$$

i. Knowing that human labour, from 2019 to 2020, is expected to grow by 0.2%, how much of projected real output growth for 2020 is explained by the factors of production, and how much is explained by the multiplier term A? What is one name usually given to this multiplier? (Note: all money variables must be in real values, not nominal).
ii. A team of researchers at the Magneto School of Economics have been studying the possible links between energy consumption – at the *primary*, *final*, and *useful* stages of energy flows – and economic growth. They have concluded that useful energy consumption is closely related with economic growth. Define, giving examples, each of the three stages of energy flows, and briefly discuss why useful energy could be the correct measure to understand the relationship between energy use and economic growth.

9) (EARN Exam 04.02.2020) An emissary is sent from Anarres to Urras, to learn from this capitalist society. He discovers that the Urrasti have implemented consistent accounting techniques to measure their nation's economic activity. Here is data available for 2017, in real terms (ROW – Rest Of the World):

Household savings: 324
Government consumption: 202
Taxes on production: 80
Borrowing from ROW: 256
Income payments from ROW: 68
Consumption of fixed capital: 507
Current net transfers from ROW: 43
Income payments to ROW: 103

Taxes on consumption: 230Total consumption: 922

a) If the Urrasti government gives 224 in subsidies to stimulate production but no subsidies to households, is there a government surplus or deficit? Explain.
b) Knowing that, in 2017, the trade balance was -311, determine how much Urras lends to the ROW in that year, and how much is invested in production.
c) Compute GDP, GNI, GNDI, Gross Savings and Net Savings. Give one example of income payments to ROW and income payments from ROW.
d) The production function for the Urrasti economy is given by

$$GDP = [a \cdot K^{\rho} + (1 - a) \cdot L^{\rho}]^{1/\rho}$$

i) Does this function have constant returns to scale? Explain.
ii) Complete the table below. (Note: assume $\rho = 1$, and $a = 0.4$).

Year	Observed (nominal) GDP	CPI	Inflation rate (%)	Observed (real) GDP	K (real)	L	Estimated (real) GDP	TFP
2016		100		1100	476	675		
2017		110		1130	479	686		
2018		130		1157	482	692		1.903

iii) A study commissioned by the Urrasti government, suggested that rises in energy efficiency might provide an explanation for the gap in observed/estimated output. The researchers identify the following relationship between energy efficiency (EFF) and TFP:

$$TFP = 0.851 \cdot \left(\frac{EFF}{0.11}\right)$$

For 2017, energy efficiency was approximately 24%. How much do you estimate TFP and GDP to be for that year? Does this study provide a better explanation for Urrasti economic growth?

10) (EARN Exam 09.09.2020) House Atreides has been assigned to take over management of Arrakis, the Desert Planet, from House Harkonnen. The Harkonnens have failed to implement a System of National Accounts to track money flows in the planet's economy, composed of: households, firms, government, banks, and the rest of the universe (ROU). For the last fiscal year of 2018, only the following information is available (in nominal terms):

Private consumption:131.9
Total consumption: 166.5
Net primary income flows to ROU: -5
Net taxes on private consumption: 13.6
Consumption of fixed capital:35.5
Lending: 89.3
Borrowing: 88.4
Investment: 35.8
Current net transfers from ROW: 4.6

a) What level of (net) taxes on production would prevent a government deficit? Are firms paying more in taxes than they receive in subsidies? Explain.
b) Assuming that the government deficit is zero, determine the GDP for 2018, using the expenditure and output approaches (identify which is which). Compute GNI, GNDI, and Net Savings, explaining how these quantities differ from GDP.

c) The Harkonnens reported a growth of GDP of 5% between 2017 and 2018, but did not take into account inflation. Knowing that the CPI for 2018 is 102, and that 2017 is the base year, how much did the economy grow, in real terms?

d) Duke Leto has tried to describe the GDP in Arrakis as a function of the factors of production capital K and human labour L, in the form of an aggregate production function (APF) such as:

$$Y = K^{0.3} \cdot L^{0.7}$$

i. Knowing that the real capital stock in 2018 was 543.1, use the perpetual inventory method to determine the real capital stock in 2017 – use the same CPI as in c).

ii. If human labour L grows by 2% between 2017 and 2018, how much of GDP growth, in real terms, is explained by the APF? And how much did total factor productivity grow in that period? Use a growth accounting approach.

iii. Scientists at the service of Duke Leto have found a relationship linking final-to-useful exergy efficiency EFF with total factor productivity TFP:

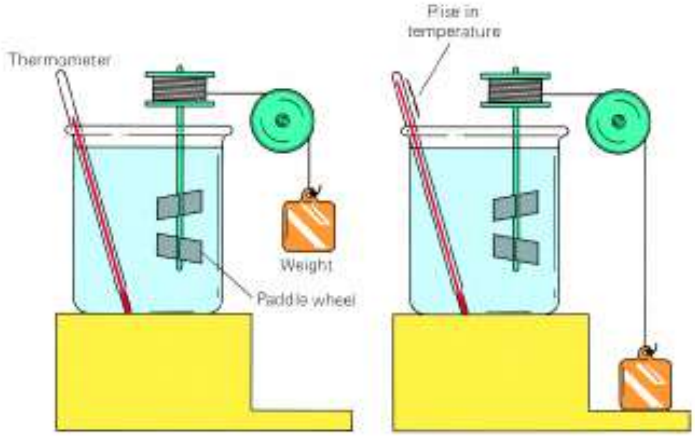
$$TFP = 1.05\left(\frac{EFF}{0.12}\right)$$

Explain, in your own words, what is meant by final-to-useful exergy efficiency, and determine how much would EFF have to grow between 2017 and 2018 in order to fully explain TFP change.

Energy

[For the multiple choice questions, choose the correct answer(s) or indicate that all possibilities are wrong, and, in all cases, give a short justification.]

1) (SDEE-D Exam, 29.01.2019). Consider a car travelling from Lisbon to Oporto, in two situations: (a) reaching a maximum speed of 120 km/h; (b) reaching a maximum speed of 100 km/ha. Describe the energy transformations, exergy destruction and entropy production happening in both cases, distinguishing what is different because of the different maximum speeds.
2) (SDEE-D Exam, 28.01.2019). Consider the Joule experiment, going from the initial situation on the left to the final situation on the right.



a) Starting from the initial moment when the weight is dropped, describe all the energy transformations that happen in this experiment, and whether and how they produce entropy.
b) Explain why the Joule experiment expresses a certain equivalence between internal energy (informally, heat) and work.
c) Explain in what sense heat and work are not fully equivalent.
d) How is the contrast between your answers to questions a) and b) related to the first and second laws of thermodynamics?
e) What is the correct measure to add up heat and work that considers that they are not of equal value? Why is it the correct measure?

3) (SDEE Exam, 09.01.2018). Should we heat a house using a natural gas boiler or an electrical resistance?

   a) With an electrical resistance because its 1st law efficiency is higher than that of a natural gas boiler.

   b) With an electrical resistance because its 2nd law efficiency is higher than that of a natural gas boiler.

   c) It depends on how the electricity was generated.

4) (SDEE Exam, 07.01.2016). In a given chain of energy flow, final energy is always lower than primary energy because:

   a) of the 2nd law of thermodynamics;

   b) of the 1st law of thermodynamics.

5) (SDEE Exam, 31.01.2018). In a given chain of energy flow, final exergy is always lower than primary exergy because:

   a) of the 2nd law of thermodynamics;

   b) of the 1st law of thermodynamics;

   c) because 1st law efficiencies are always lower than 1;

   d) because 2nd law efficiencies are always lower than 1.

6) (SDEE Exam, 01.02.2016). In a given chain of energy flow, useful energy is:

   a) always lower than final energy because of the 1st law of thermodynamics;

   b) always lower than primary energy because of the 1st and 2nd law of thermodynamics;

   c) maybe higher than final energy.

7) (SDEE-D Exam 09.01.2018). In a given chain of energy flow, final exergy is always lower than primary exergy because:

   a) of the 2nd law of thermodynamics;

   b) of the 1st law of thermodynamics;

   c) 1st law efficiencies are always lower than 1.

8) (SDEE-D Exam 09.01.2018). Explain how the Joule experiment expresses the 1st and the 2nd Laws of Thermodynamics.

9) (SDEE-D Exam, 31.01.2018) Exergy is a preferred method to add up heat and work because: [indicate and justify, for each option, whether it is right or wrong]

a) 1J of heat can at best be transformed into less than 1 J of work;

b) When we burn a substance, the heat we obtain is much lower than its exergy;

c) Exergy adds up all energy flows in a common reference, the maximum potential to provide work.

10) (SDEE-M Exam 18.01.2018). Explain the advantages of using second law efficiencies when compared to first law efficiencies. (300 words max.)

11) (SDEE-M Exam 30.01.2018). Explain the relations:

a) between first law efficiencies and the first law of thermodynamics;

b) between second law efficiencies and the second law of thermodynamics;

c) between second law efficiencies and exergy.

d) because 2nd law efficiencies are always lower than 1.

12) (SDEE-D Exam 15.01.2019). Consider the following facts:

Natural gas, used, e.g., in power plants and for cooking at home, is methane, CH4.

Ruminants (cattle, sheep, etc.), in their process of digesting mostly grasses which are inedible for humans, release methane.

Answer the following questions, providing a justification:

a) Which has higher exergy, methane or carbon dioxide?

b) Suppose you burn 1J of methane, with two outcomes: a heat flow at 1000 K; a heat flow at 500 K. Which has higher exergy?

Discuss the following statement

c) "Eating meat from ruminants is inefficient, because, due to the 2nd law of thermodynamics, exergy is destroyed in the processes going from their ingestion of feed to their building up body mass for our consumption."

13) (SDEE-D Exam 15.01.2019). What factors may influence exergy efficiency and final exergy consumption when making a long-run societal exergy accounting?

Energy, the Biosphere and Economic Growth

[For the multiple-choice questions, choose the correct answer(s) or indicate that all possibilities are wrong, and, in all cases, give a short justification.]

1) (SDEE Exam, 11.01.2017). Discuss the merits of measuring energy flows in an economy using the exergy metric, considering the different stages of energy transformation in an economy.

2) (SDEE Exam, 11.01.2017). Roughly speaking, in an economy labour obtains a 2/3 share of the income from economic production and capital obtains a 1/3 share. Explain, covering comprehensively the relevant content of this course, what this means regarding the role of capital and labour in economic growth, and what else is needed to understand economic growth.

3) (SDEE Exam, 31.01.2017). Describe the thermodynamic view on human activity in the biosphere, and how it relates to the concepts of "empty-world economy" vs. "full-world economy" (or "cowboy economy" vs. "spaceship economy").

4) (SDEE Exam, 31.01.2017). In Portugal, in the last 150 years, the ratio of useful exergy to GDP has been roughly constant. Discuss the implications of this regarding the reduction in carbon dioxide emissions in the future.

5) (SDEE Exam, 18.01.2018). Discuss comparatively the relation between the primary and useful stages of energy transformation in societies and the creation of economic value. (300 words max.)

6) (SDEE Exam, 18.01.2018). Give and describe an example of an energy transformation technology that increases the productivity of labour in firms. (300 words max.)

7) (SDEE Exam, 18.01.2018). Using the material of this course, discuss critically the statements

   a) "pollution generated by human activities is an unavoidable consequence of the laws of thermodynamics" (150 words max.);

   b) "the laws of thermodynamics impose a maximum size on human activity on Earth". (150 words max.).

8) (SDEE-M 30.01.2018) Write a concise text that covers in an integrated way the following topics (450 words max.):

   a) What is the relation between capital, labour and economic growth?

   b) Why are capital and labour not enough to explain economic growth?

   c) Why is useful exergy the correct measure of energy flow to understand the relation between energy and the creation of economic value?

   d) How does useful exergy help in explaining economic growth?

9) (SDEE-D Exam 09.01.2018). Economic growth happens because

   a) entrepreneurs find ways of combining technological developments and management innovation in new business models;

   b) the real cost of capital is decreasing along time;

   c) a fixed amount of useful exergy is required per unit of economic value;

   d) the amount of capital increases along time.

10) (SDEE-M Exam, 12.01.2019).

    "If we find a strong correlation between exergy usage and economic growth, we can conclude that TFP, or the Solow residual, has finally been explained."

    Do you agree with this statement? Explain your answer.

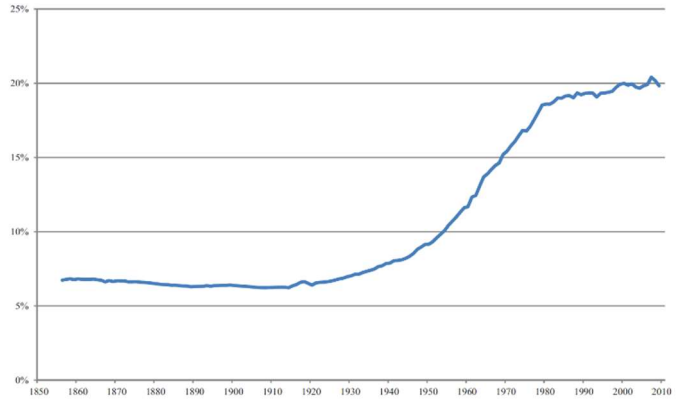
11) (SDEE-D Exam, 29.01.2019). Does increasing the energy efficiency reduce the amount of energy used? Consider the strong correlation between exergy usage and economic growth. Present at least one measure to effectively reduce energy use in this context.

12) (SDEE Exam, 30.01.2018). It is now cheaper to buy an engine that delivers 100 kW than it was 100 years ago. Explain the implications of this for the process of economic growth.

13) (SDEE-M 30.01.2018) Describe the four industrial revolutions, their defining technologies and the associated energy transitions. (300 words max.)

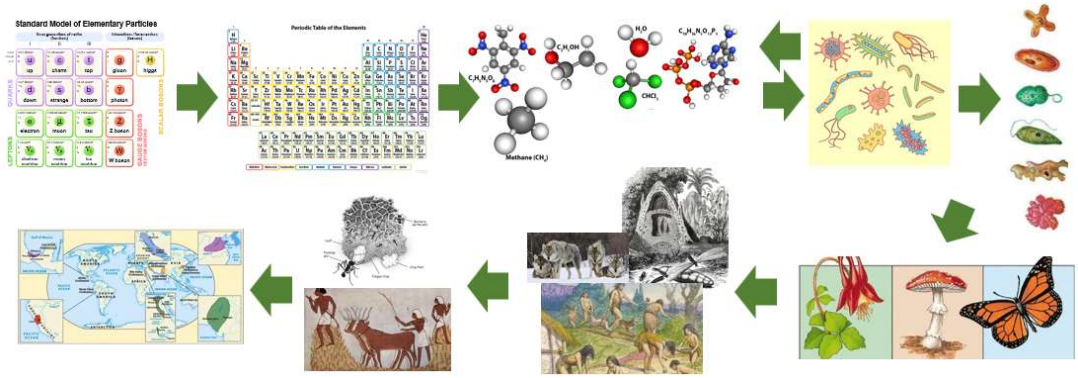
14) (SDEE-D Exam, 30.01.2018). Explain how the control of energy at successively smaller scales was relevant for increasing economic productivity and how the fourth industrial revolution continues this trend.

15) (SDEE-M Exam, 28.01.2019) The following figure shows the final to useful exergy efficiency of Portugal 1856-2009 (Serrenho et al. 2016). Efficiency ranges from 7% to 20% approximately. Describe what may influence final to useful exergy efficiency in societal exergy accounting, considering this example from Portugal and/or the analogous example for the country you studied in your course project.



16) (SDEE-M Exam 28.01.2019). Does increasing the energy efficiency reduce the amount of energy used? Consider the strong correlation between exergy usage and economic growth. Present at least one measure to effectively reduce energy use in this context. If relevant, present examples or insights from your course project.

17) (EARN Exam 17.01.2020) Think of primary energy primary sources (e.g., oil, coal, nuclear, biomass). Choose one and discuss it with a **Big History perspective**, i.e., write a brief description of the history of your chosen primary source from the Big Bang based on the **kits of building blocks** (shown below) that summarize paths of the complexification we see in the universe. In particular, make explicit reference to **specific elements** of these kits and describe their importance for the primary energy source you are discussing (max. 150 words).



Sustainable Development and Sustainability Indicators

1) (SDEE Exam, 09.01.2018). Sustainable development is commonly translated as considering three pillars: economic, social and environmental. As illustrative examples consider economic growth for the economic pillar, equity for the social pillar and carbon emissions for the environmental pillar. Considering these three

issues discuss the opportunities and threats for sustainable development of the fourth industrial revolution that is now starting. (max. 300 words).

2) (SDEE Exam, 07.01.2016). The ecological footprint, as currently calculated,
   a) shows that the current activity of humanity is unsustainable because every year we use 1.5 time the area of the Earth;
   b) show that the current activity of humanity is unsustainable if we assume that the majority of CO2 emissions have to be removed from the atmosphere by planting forests;
   c) is meaningless, because it implies assuming that we currently use 1.5 times the area of the Earth.
3) (SDEE Exam, 01.02.2016). Regarding the ecological footprint, considering the technology of concentrating solar power is relevant because:
   a) it is a technology which provides us with very large amounts of energy and thus shows that there are no limits to growth;
   b) it is a technology which is more efficient, in terms of area, than forests at removing carbon from the atmosphere or avoiding carbon emissions;
   c) it is a technology that can be deployed in deserts.
4) (SDEE Exam 11.01.2017). The ecological footprint considers, among other items, the carbon footprint, i.e., the potential forest area required to compensate for the carbon emissions of a country. For the purpose of this calculation, discuss whether the carbon emissions, a.k.a. "carbon responsibility", of a country should be calculated using the territorial (a.ka. direct), consumption (a.k.a. upstream) or income (a.k.a. downstream) perspectives.
5) (SDEE-M Exam 18.01.2018)
   a) Is the ecological footprint an indicator of whether we are in an "empty-world economy" or in a "full-world economy"? (150 words max.)
   b) Discuss the advantages and disadvantages of the current calculation method for the ecological footprint. (150 words max.)
6) (SDEE-M 30.01.2018) Describe what improvements you would make to the ecological footprint to correct its deficiencies. (200 words max.)

Integrative Questions / Projects

1) (SDEE Exam 11.01.2017). Consider the first and second laws of thermodynamics, the foundation of the approach taken in this course to the analysis of sustainable development issues.
   a) Discuss whether they are falsifiable, in the sense of Karl Popper's philosophy.
   b) Describe how they establish constraints on the process of economic growth.
1) Using the knowledge gathered from the following readings:
   - Funtowicz and Ravetz
   - Sandel,
   - Class material on uncertainty and philosophy of knowledge
   - Class material on discounting

- The Stern Review (2007), pp. 23-54.
- Nordhaus, W. (2007). Critical assumptions in the Stern review on climate change. *Science* 317: 201-202.
- Fisher, Norgaard, Howarth, Nordhaus (2007) CO2 emissions - getting bang for the buck. *Science* 318: 1865-1868.
- Spash (2007). The economics of climate change impacts à la Stern. *Ecological Economics* 63: 706-713.
- Heal, G. (2008). Climate economics: A meta-review and some suggestions. *NBER Working Paper* No. 13927.

Take as your departing point pp. 15-19 in the Hartwell paper and carry out a rough assessment of the climate change debate, considering:

- the uncertainties and the stakes involved;
- what libertarianism, utilitarianism, Kant and Rawls have to say about burden sharing (intra and inter-countries) in addressing climate change;
- the debate on discounting and climate change, applying the knowledge acquired in the lectures on discounting.

Other instructions:

- In the text, reference all content.
- Insert at the end all references used, using a standard formatting (choose the formatting from any scientific journal).
- Max. 8 page text.